Magnetization loop modelling for a superconductor/ferromagnetic composite

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Magnetization loop modelling for a superconductor/ferromagnetic composite of a magnetic cloak (composite = 1SC/1FM)

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Outline

1) Introduction
2) Numerical model
3) Experimental verification
4) Conclusions
Introduction

magnetic invisibility cloak: our ultimate goal

no distortion of the applied field outside the shielded space
magnetic cloak: more than a simple shield

combination of diamagnetic and ferromagnetic material

\[ \mu_r = \frac{R_{Fo}^2 + R_{Fi}^2}{R_{Fo}^2 - R_{Fi}^2} \]

no distortion of the applied field outside the cloak

C. Navau, D.-X. Chen, A. Sanchez, N. Del-Valle,
DC Cloak (#1)

**SC tape** (Superpower 12 mm wide, 2 turns)
**Fe9Ni18Cr sheet** (0.1 mm thick, 5 turns)

mapping of magnetic field (Hall probe)

calculation

![Graph showing the mapping of magnetic field for FM only, SC/FM cloak, and SC only with horizontal and magnetic field coordinates.]

experiment

![Graph showing the mapping of magnetic field for FM only, SC/FM cloak, and SC only with horizontal and magnetic field coordinates.]

cloaking properties confirmed

Finite element calculations in 2D

**superconductor:** $J(A, \nabla \varphi)$ formulation

“smoothed” critical state model

$$J = J_c \tanh \left( \frac{E}{E_c} \right)$$

zero net transport current achieved through electrostatic charges:

in the neutral zone

$$E_z = 0$$

$$\frac{\partial A_z}{\partial t} = -\nabla \varphi$$

$$J(x, y)|_i = J_c \tanh \left( -\frac{1}{E_c} \frac{\partial [A(x, y) - A_{0,i}]}{\partial t} \right)$$
Finite element calculations in 2D

ferromagnetic material: magnetic permeability

\[ \mu_r \]
Finite element calculations in 2D

ferromagnetic material: magnetic permeability

AC regime: AC loss because of magnetic hysteresis

FM loss (per unit length)

$$Q_{FM} = \int_{FM} Q(B_{max}(x, y))dx\,dy$$


FM loss is added to the SC loss to obtain the total loss.
Cloaking of AC fields

new phenomenon: irreversibility of $m(B)$
→ dependence on history
→ phase shift of induced magnetic field
→ AC loss

$$M = m/V_{\text{cloak}}$$

$$(\chi = \mu_0 M/B)$$

$$\chi' = \mu_0 m'/ (V_{\text{cloak}} B_{\text{max}})$$

$$\chi'' = \mu_0 m'' / (V_{\text{cloak}} B_{\text{max}})$$

AC susceptibility

$$\frac{Q}{V_{\text{cloak}}} = \pi \chi'' \frac{B_{\text{max}}^2}{\mu_0}$$
Cloaking of AC fields

SC tape (Superpower 12 mm wide, 4 strips)

Cloak A: Fe9Ni18Cr sheet (0.1 mm thick, 5 layers)
Cloak B: Fe-Si-B-Cu-Nb metallic glass (0.025 mm thick, 4 strips)
Properties of FM materials

Fe9Ni18Cr sheet

Fe-Si-B-Cu-Nb ribbon

Fe-18%Cr-9%Ni sheet

Fe-Si-B-Cu-Nb ribbon

10 mm

Cloak A in AC field

\[ f = 36 \text{ Hz} \]
Estimation of FM hysteresis loop

basic idea: Rayleigh’s approach \( Q \propto B_{\text{max}}^3 \)


magnetic moment (per unit length) of FM:

\[ m_{FM} = m_{\text{rev}} + m_{\text{hys}} \]
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magnetic moment (per unit length) of FM:

\[ m_{FM} = m_{rev} + m_{hys} \]

\[ m_{hys} = \pm \gamma (B_{ac}^2 - B_{\text{max}}^2) \]

\[ Q_{FM} = \int m_{FM} dB_{ac} = 2 \int_{-B_{\text{max}}}^{B_{\text{max}}} \gamma (B_{ac}^2 - B_{\text{max}}^2) dB_{ac} = \frac{8}{3} \gamma B_{\text{max}}^3 \]

\[ \gamma = \frac{3}{8} \frac{Q_{FM}}{B_{\text{max}}^3} \]

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\[ \gamma = Q_{FM} \frac{3}{8} \frac{1}{B_{\text{max}}^3} \]

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Cloak A in AC field

-0.4 -0.2 0 0.2 0.4
-0.005 -0.0025 0 0.0025 0.005
m [Am]
B [T]
m_FM m_SC m_Cloak

-1 -0.5 0 0.5 1
-0.012 -0.006 0 0.006 0.012
m [Am]
Bac [T]
m_FM m_SC m_Cloak

-4 -3 -2 -1 0 1 2 3 4
-0.075 -0.05 -0.025 0 0.025 0.05 0.075
m [Am]
B [T]
m_FM
m_SC
m_Cloak

\( f = 36 \text{ Hz} \)

FM hysteresis dominates
Cloak B in AC fields

![Graph showing chi', chi'' vs Bmax]
Cloak B in AC fields
Cloak C = Cloak B + additional SC layer

SC tape (Superpower 12 mm wide, 4 strips)
Fe-Si-B-Cu-Nb ribbon (0.025 mm thick, 4 strips)
SC tape (Superpower 4 mm wide, 9 strips)
Cloak C = Cloak B + additional SC layer

very good performance at $B_{\text{max}} < 1$ mT
Conclusions

Numerical modeling for magnetic properties of SC/FM composites in 2D provides reasonable predictions.

Adding the approximation of FM hysteresis to the calculated loops allows to understand better the magnetic behavior.

Design of cloaks reducing the AC magnetic signal 20x is possible.
AC field cloaking

Experimental apparatus

\[ B_{ac} = B_{\text{max}} \cos(\omega t) \]
\[ \omega = 2\pi f, \ f \in (36 \div 288 \text{ Hz}) \]

Magnetic hysteresis loop

AC susceptibility

\[ m \] vs. \[ B_{ac} \]
\[ \chi', \chi'' \] vs. \[ B_{\text{max}} \text{[T]} \]
Properties of FM material(s)

~ 1 meter of tape wound in toroidal sample