H-formulation reinvented: faster computations with zero air conductivity

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Contributors

- This presentation is based on two articles, currently under peer review
  - V Lahtinen, A Stenvall, F Sirois and M Pellikka *Appl. Num. Math.*
    "**A modelling tool for simulating hysteresis losses in superconductors utilizing an *H*-oriented finite element method formulation with cohomology basis functions**"
    Submitted for peer review.
  - A Stenvall, V Lahtinen and M Lyly *Supercond. Sci. Technol.*
    "**The failure of an *H*-formulation based three-dimensional hysteresis loss modelling tool in a simulation including time varying applied field and transport current–the fundamental problem and a solution with a cohomology basis function**"
    Submitted for peer review.
- A large part of this research was conducted during my research exchange period at École Polytechnique de Montréal, Canada, in the research group of professor Frédéric Sirois.
- We thank F Grilli and V Zermeno from Karlsruhe Institute of Technology (KIT), Germany, for their kind permission to use their Roebel cable mesh in this presentation.

Outline

- The H-formulation: what is there to develop?
- Another approach: zero air conductivity and cohomology basis functions
- Performance benchmark
- Conclusions

Introduction

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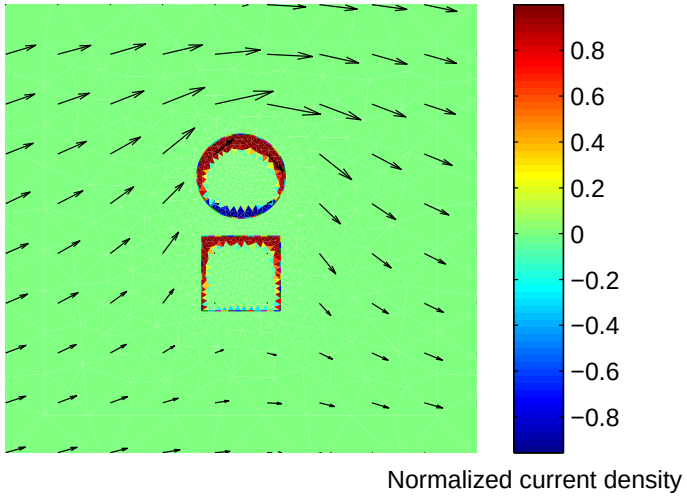

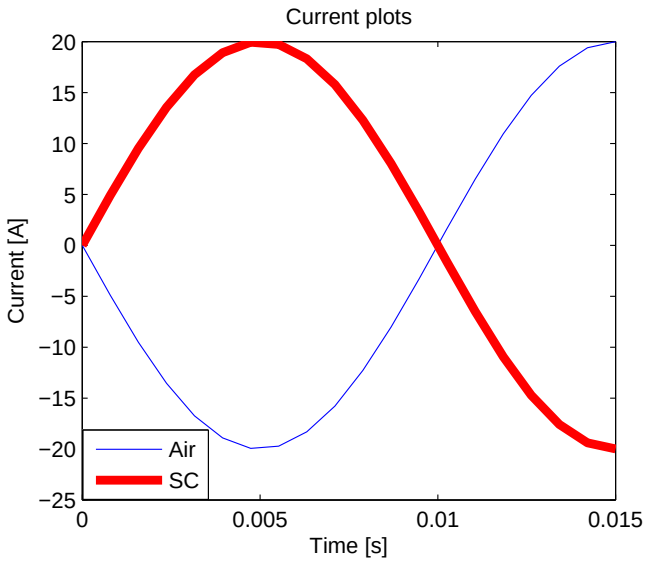

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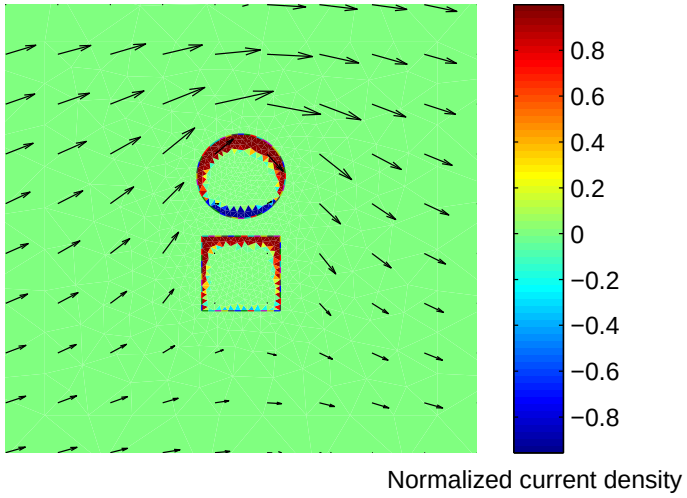
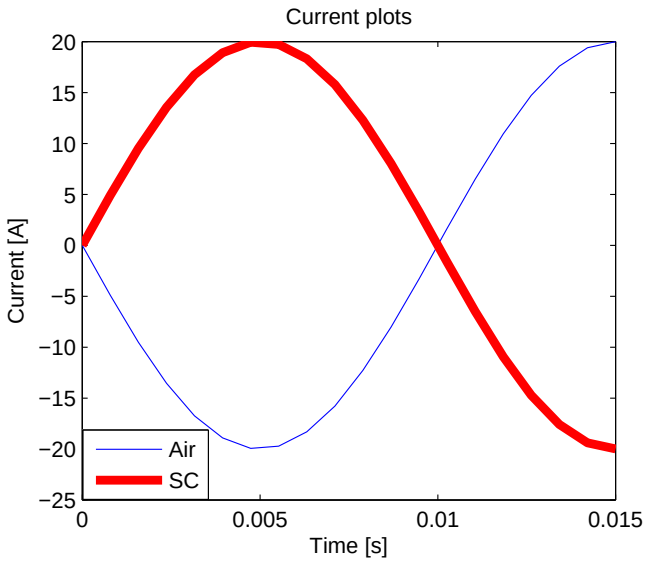
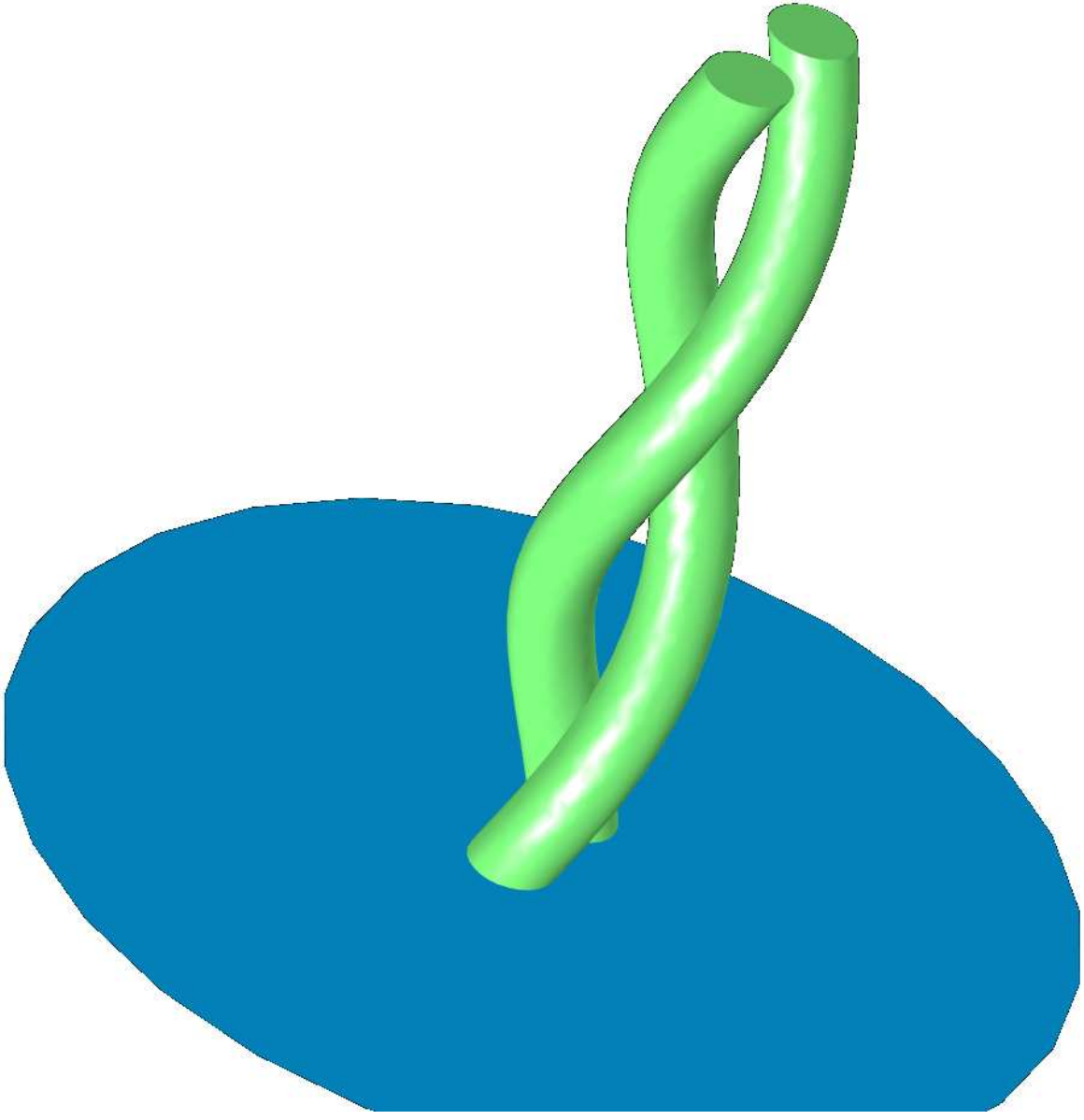




- But in 3D, things can go terribly wrong!

Failure of the H-formulation



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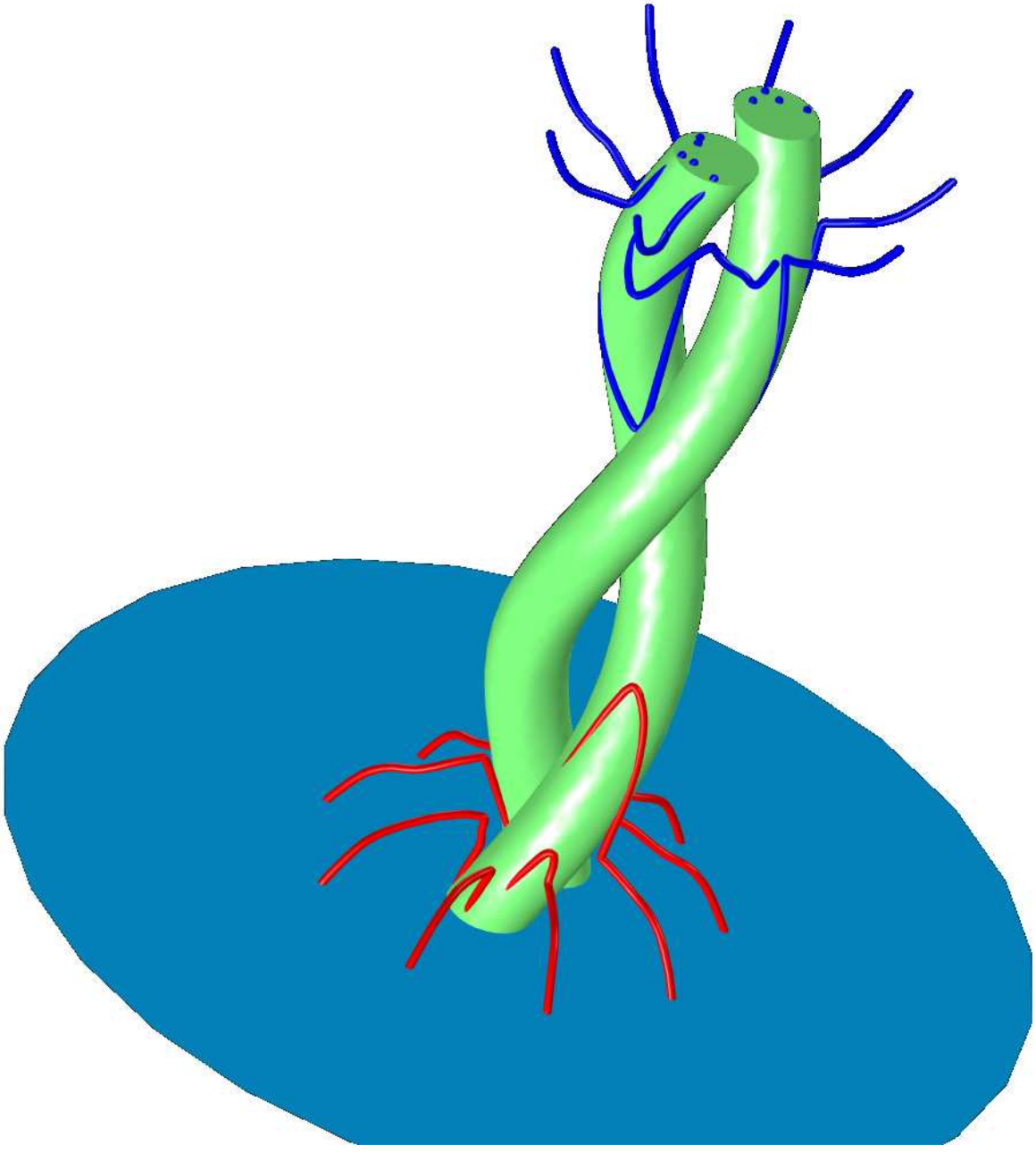

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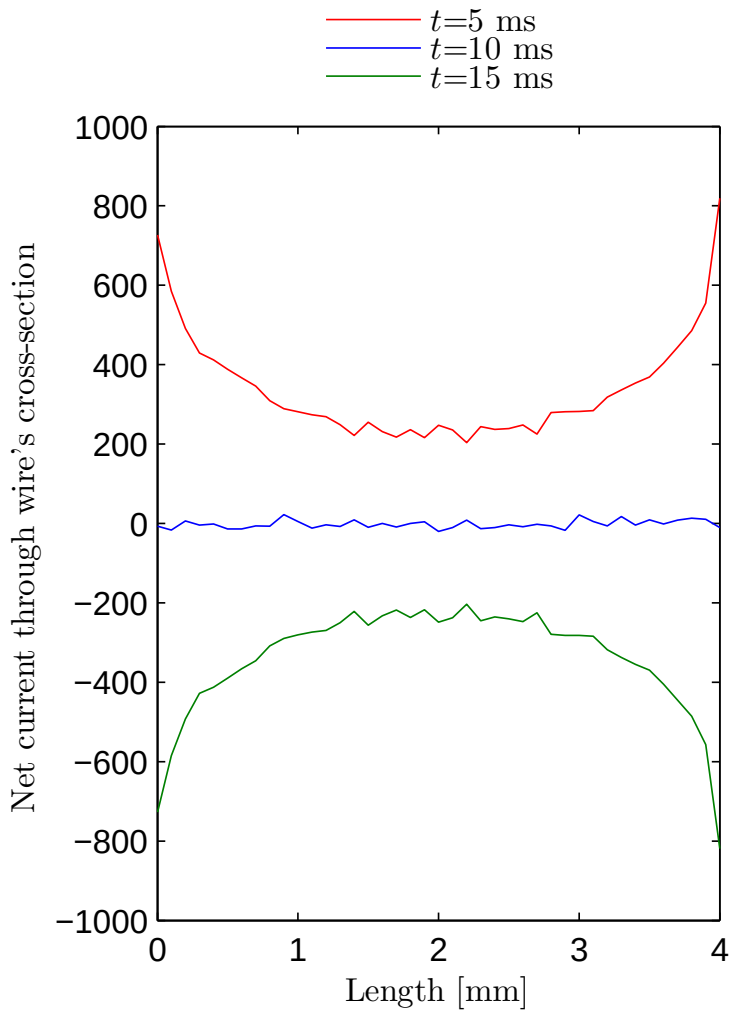

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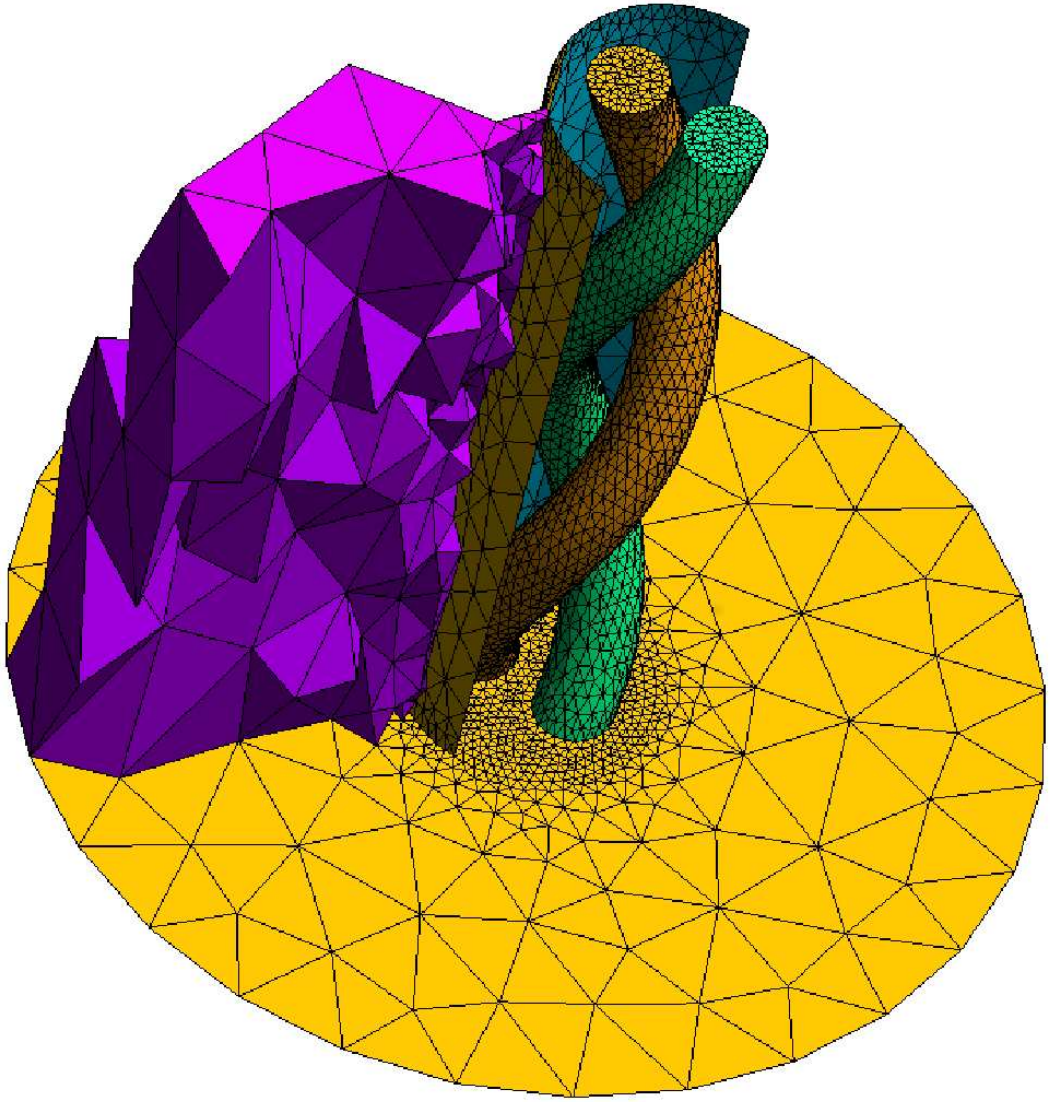
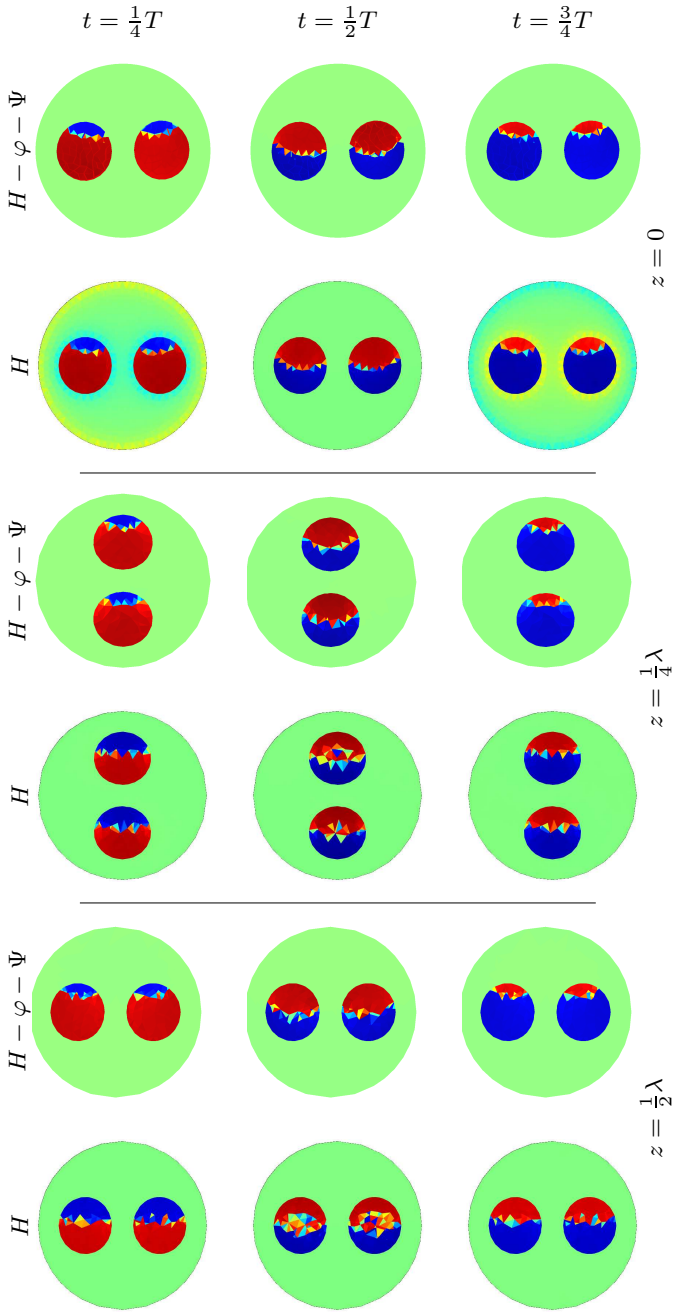
- If so, we have $\mathrm{curl}(H) = 0$ in air, but we know that whenever currents flow in the conductors, $H \neq \mathrm{grad}(\varphi)$.
- Solution: cohomology basis functions Ψ: $H = \mathrm{grad}(\varphi) + \Psi$ in the air region.

Fixing the H-formulation



Results H vs. H-φ-Ψ



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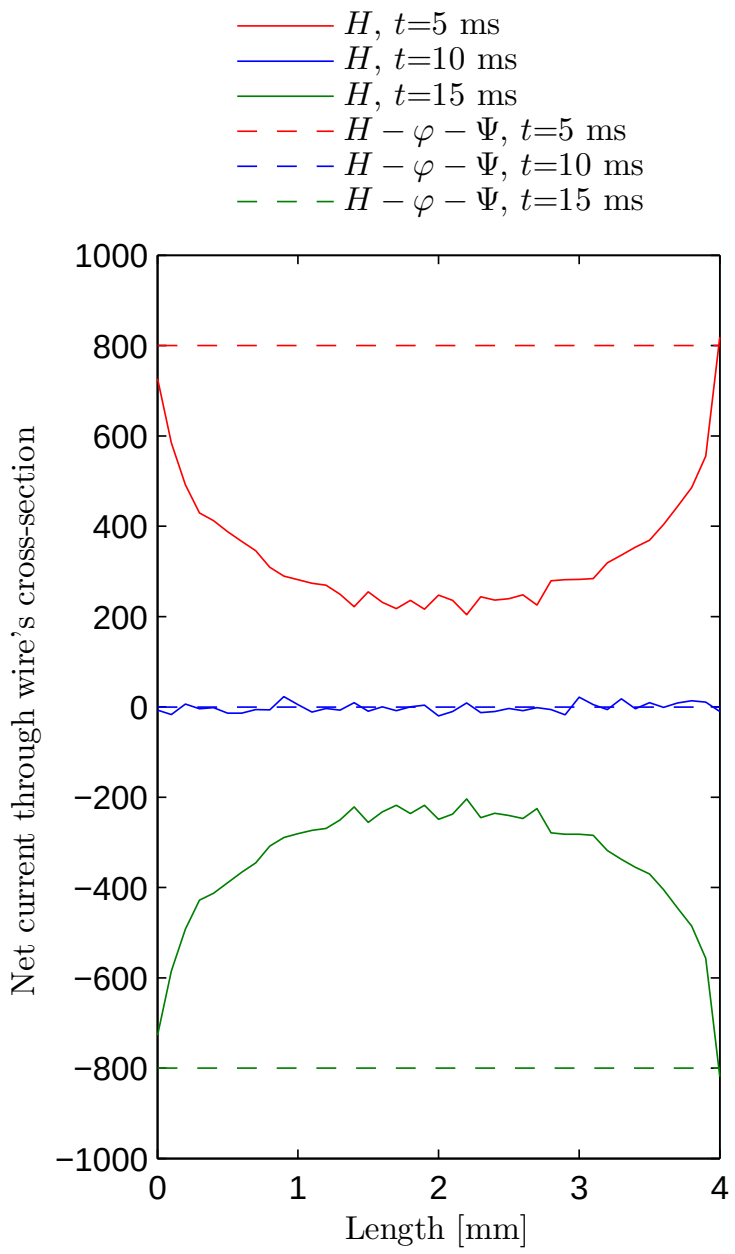

Benchmarking H vs. H-φ-Ψ

- The formulations have been implemented on the same programming framework. Exactly the same numerical parameters, such as integration and linear solver tolerances were utilized to guarantee a fair comparison.

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- Three different cases:
  - **Case 1**: 2D: Two-filament wire, filaments inside a normal conducting matrix. One current constraint. $I = 0.8 I_{\mathrm{c}} \sin(2\pi f t)$
  - **Case 2**: 2D: Four-conductor system, four current constraints + applied AC field. $I_i = 0.8 I_{\mathrm{c}} \sin(2\pi f t)$, $B_{\mathrm{ext}} = 0.1 \sin(2\pi f t)$ T
  - **Case 3**: 3D: Superconducting cylinder, one current constraint. $I = 0.5 I_{\mathrm{c}} \sin(2\pi f t)$

Benchmark results

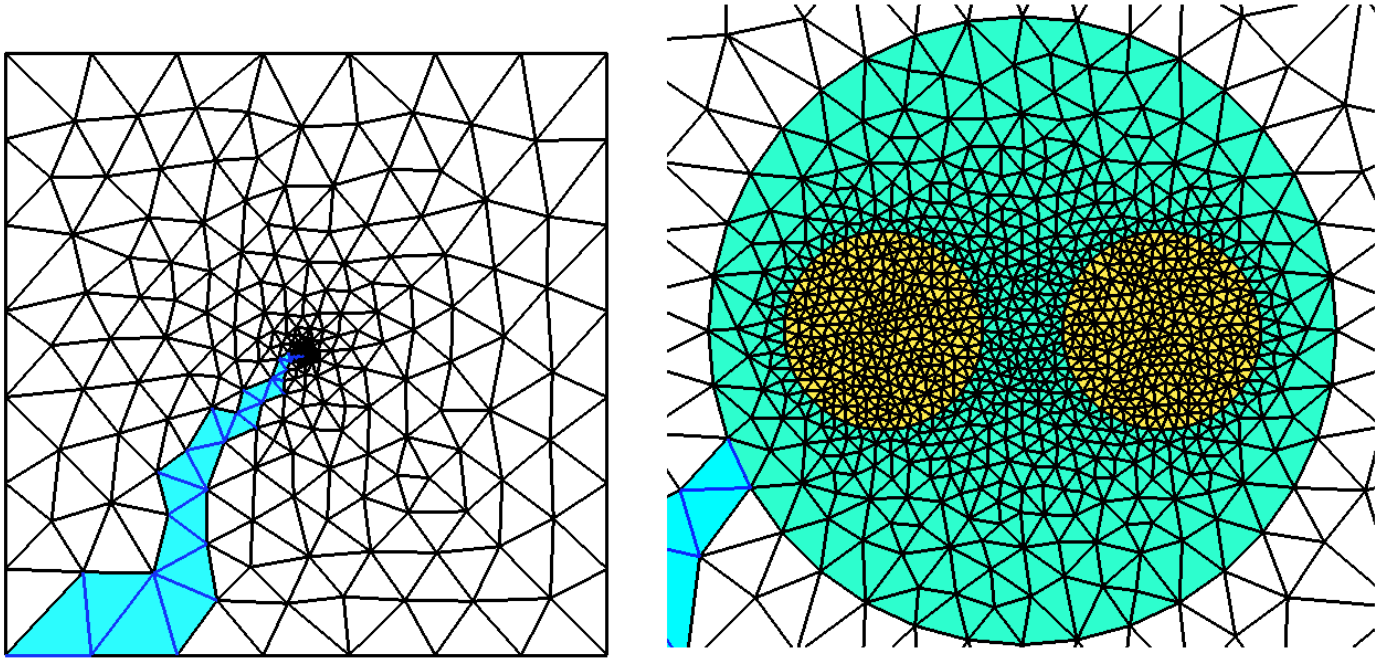

Figure: **Case 1**: The mesh and the support of the employed cohomology basis function. Decrease from H to H-φ-Ψ in **DoFs: 15 %**, in **CPU time: 25 %**.

Benchmark results

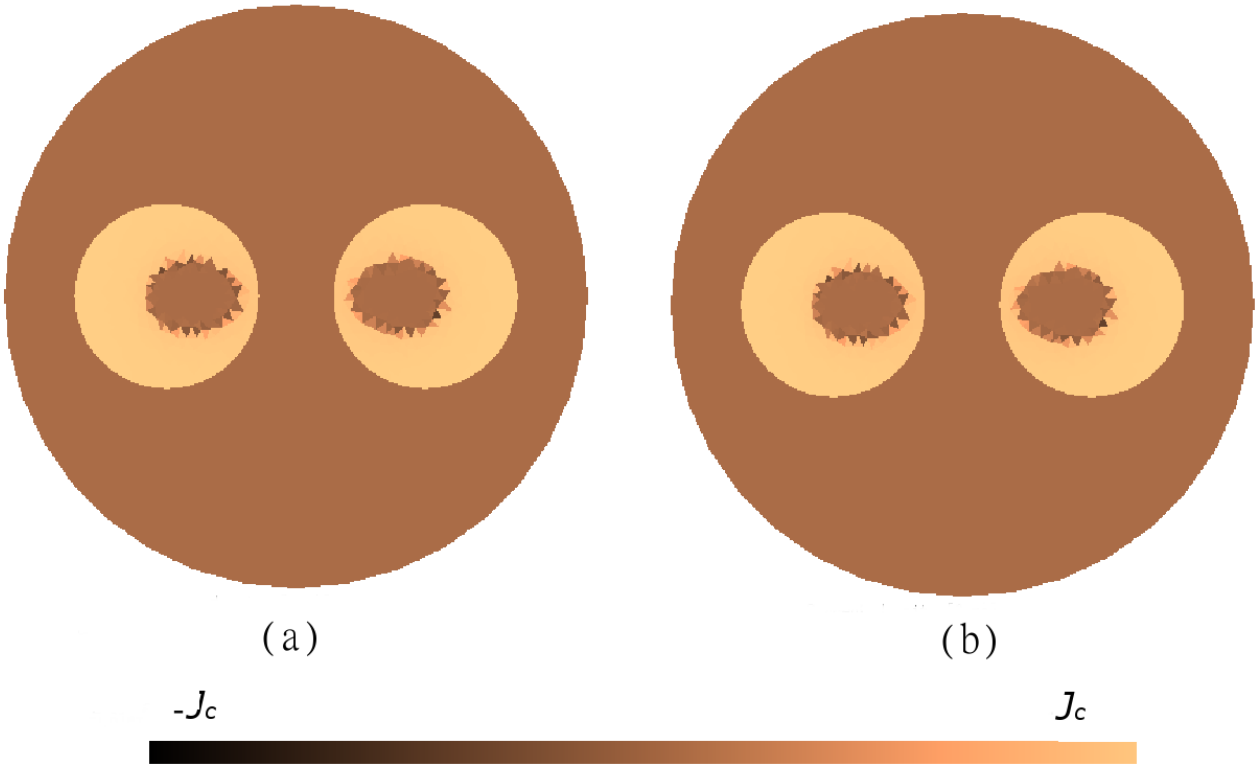

Figure: **Case 1**: The current penetration profiles for both formulations at the peak of the AC quantities. (a): H-formulation. (b): H-φ-Ψ-formulation.

Benchmark results

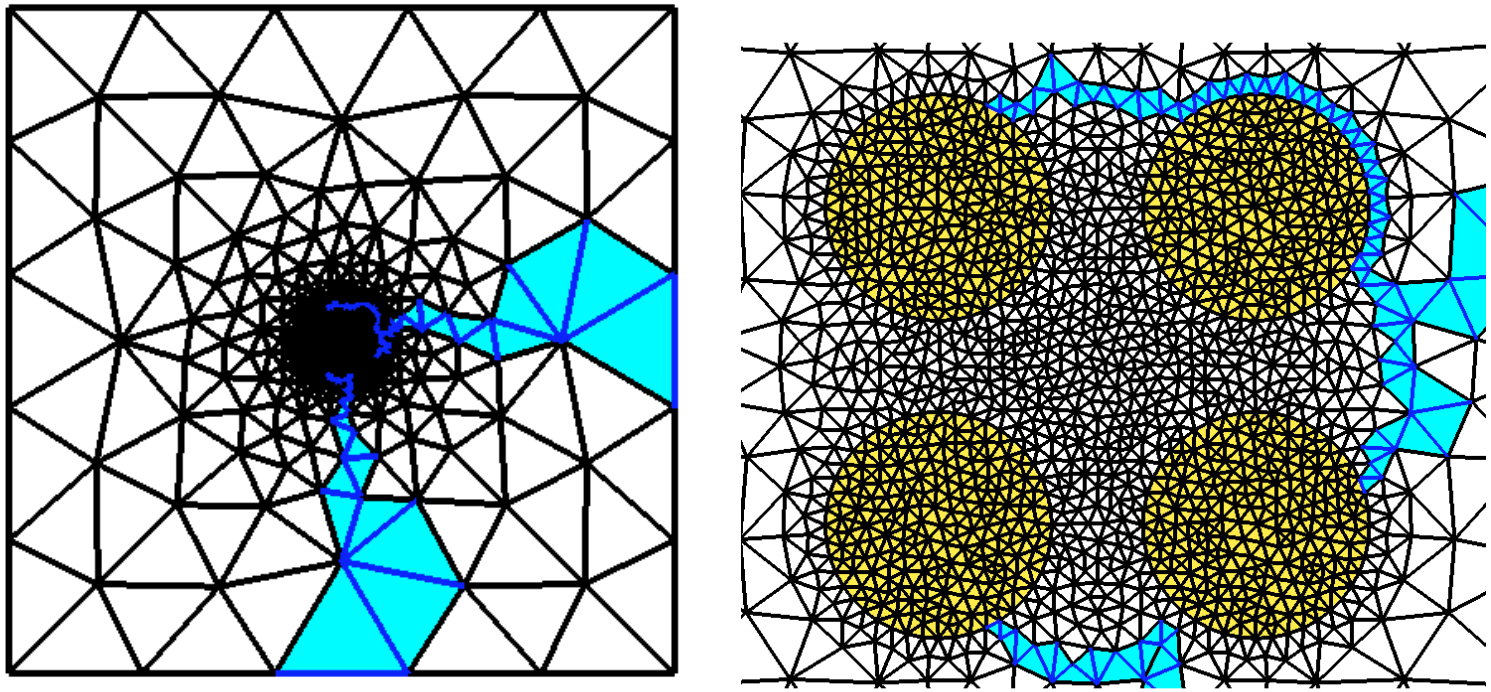

Figure: **Case 2**: The mesh and the support of the employed cohomology basis functions. Decrease from H to H-φ-Ψ in **DoFs: 34 %**, in **CPU time: 49 %**.

Benchmark results

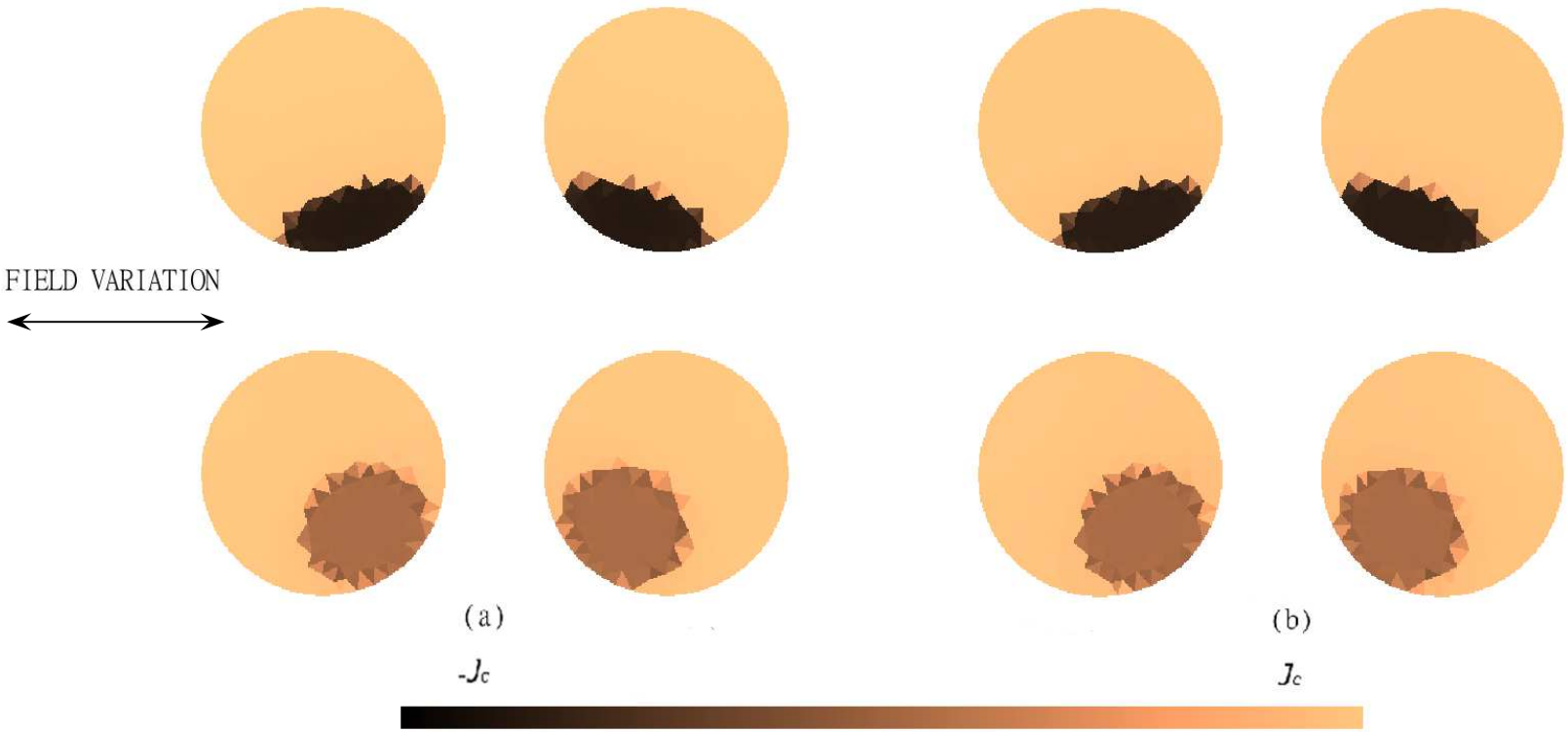

Figure: **Case 2**: The current penetration profiles at the peak of the AC quantities. (a) H-formulation and (b) H-φ-Ψ-formulation.

Benchmark results

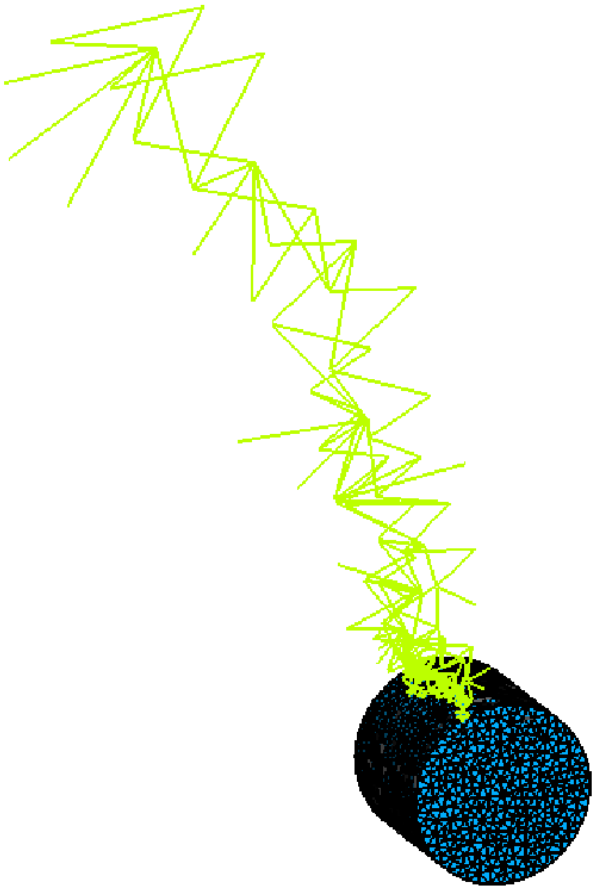

Figure: **Case 3**: The mesh and the support of the employed cohomology basis function. Decrease from H to H-φ-Ψ in **DoFs: 43 %**, in **CPU time: 23 %**.

Cohomology basis for a Roebel cable mesh

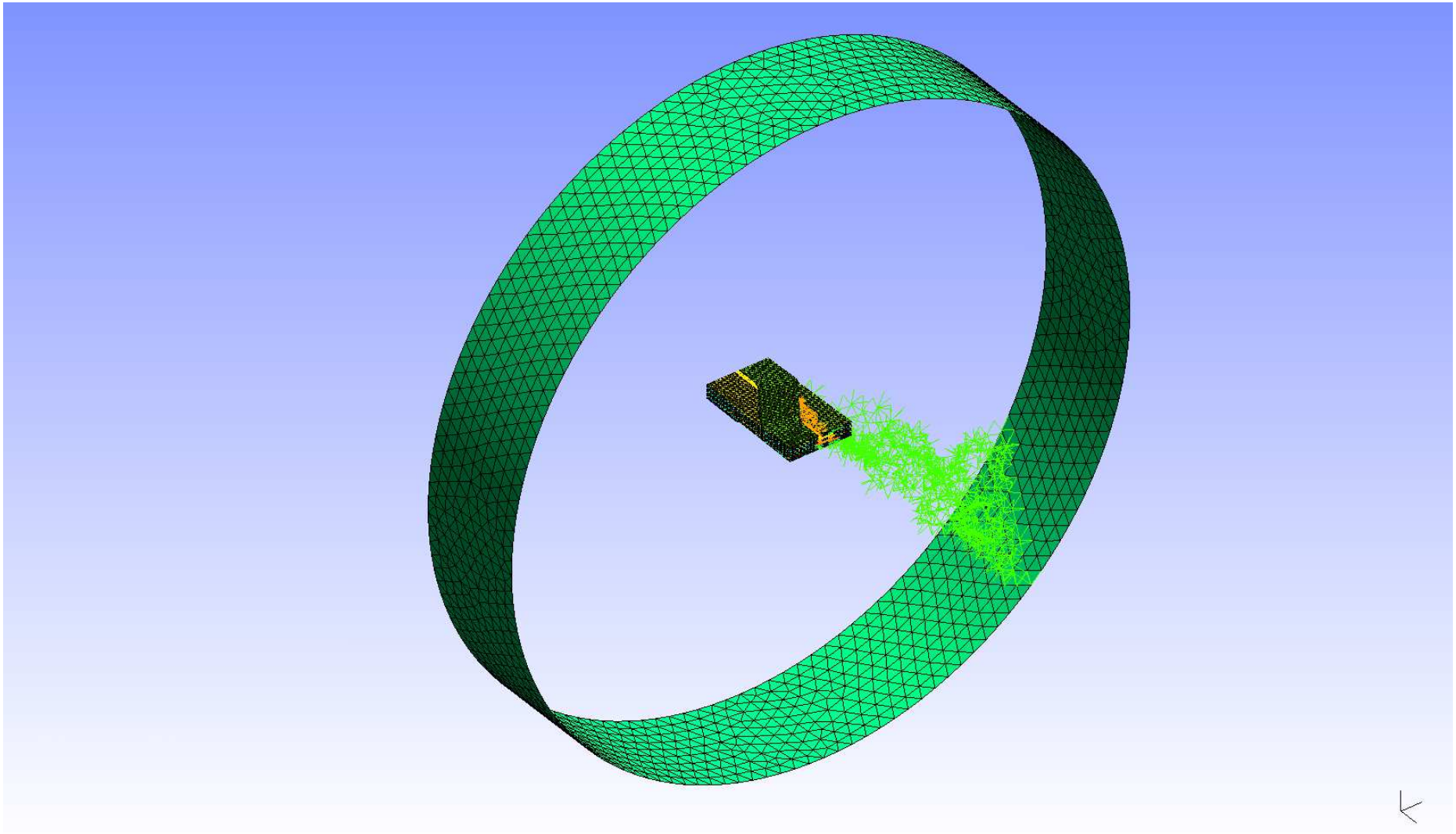

Figure: Roebel cable mesh by F Grilli and V Zermeno

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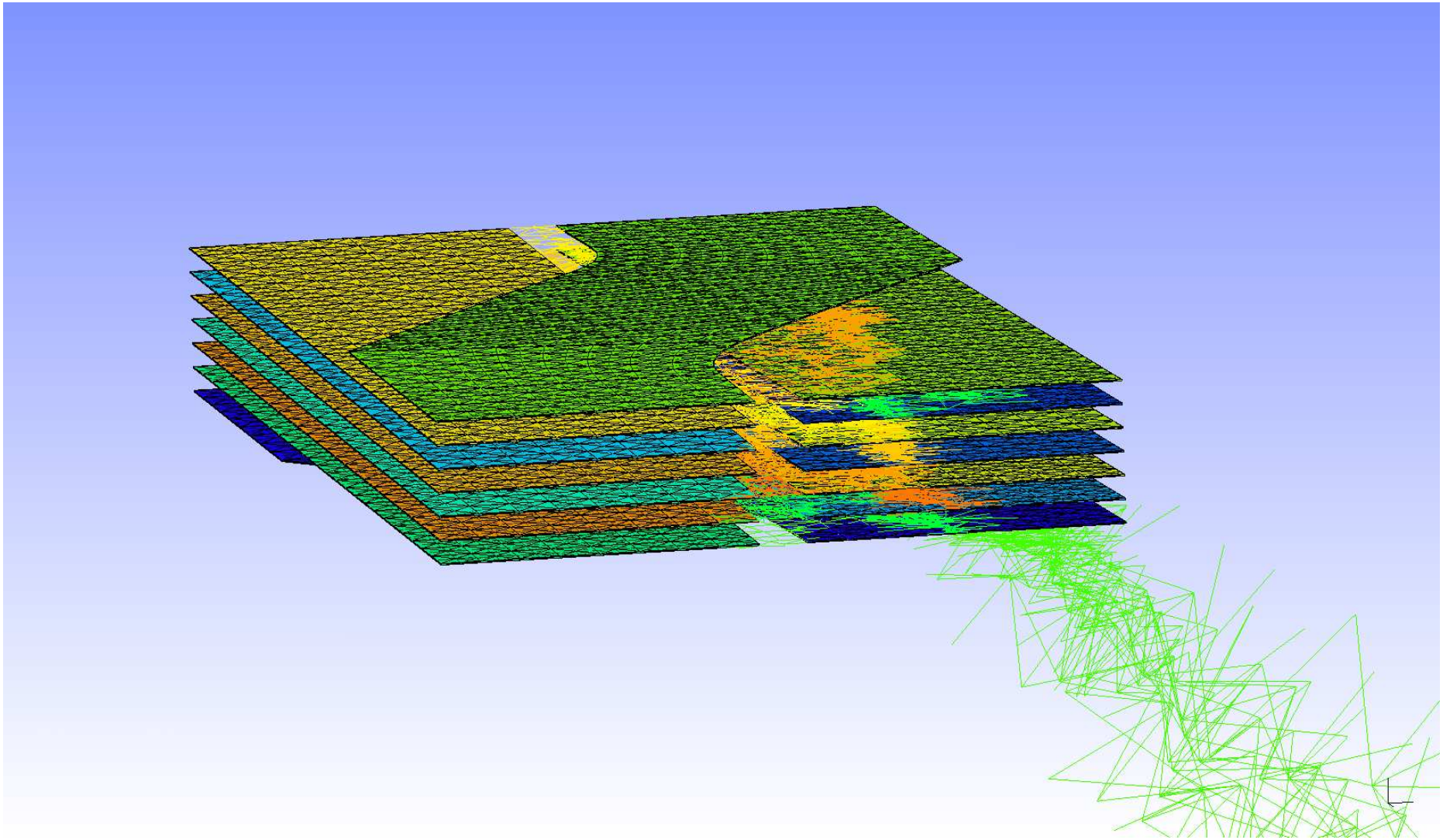

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- In AC-AC cases, utilizing finite air resistivity in the H-formulation may result in a situation that the modeller does not intend to simulate.
- This can be fixed by setting conductivity to zero in the air regions and utilizing cohomology basis functions in the approximation of H.
- The H-formulation with cohomology basis functions results in faster computations than the traditional one, when the same mesh is utilized.

Thank you