$H$-formulation reinvented: faster computations with zero air conductivity

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Contributors

- This presentation is based on two articles, currently under peer review
    "A modelling tool for simulating hysteresis losses in superconductors utilizing an $H$-oriented finite element method formulation with cohomology basis functions"
    Submitted for peer review.
    "The failure of an $H$-formulation based three-dimensional hysteresis loss modelling tool in a simulation including time varying applied field and transport current—the fundamental problem and a solution with a cohomology basis function"
    Submitted for peer review.

- A large part of this research was conducted during my research exchange period at École Polytechnique de Montréal, Canada, in the research group of professor Frédéric Sirois.

- We thank F Grilli and V Zermeno from Karlsruhe Institute of Technology (KIT), Germany, for their kind permission to use their Roebel cable mesh in this presentation.
The $H$-formulation: what is there to develop?

Another approach: zero air conductivity and cohomology basis functions

Performance benchmark

Conclusions
Introduction

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![Normalized current density](image1)

![Current plots](image2)
Introduction

- $H$-formulation utilizes finite air resistivity. $\rightarrow$ Unphysical currents in the air regions.
- Question: Can this cause problems in simulations?
- In 2D, everything is fine as long as the resistivity in the air region is high enough compared to the truly conducting regions.

- But in 3D, things can go terribly wrong!
Failure of the $H$-formulation
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![Graph showing net current through wire's cross-section as a function of length for different time intervals: $t=5$ ms (red), $t=10$ ms (blue), and $t=15$ ms (green).]
Fixing the $H$-formulation

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- Solution: cohomology basis functions $\Psi$: $H = \text{grad}(\varphi) + \Psi$ in the air region.
Fixing the $H$-formulation
Results $H$ vs. $H-\phi-\psi$
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Benchmarking $H$ vs. $H-\varphi-\Psi$

- The formulations have been implemented on the same programming framework. Exactly the same numerical parameters, such as integration and linear solver tolerances were utilized to guarantee a fair comparison.
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Three different cases:

- **Case 1**: 2D: Two-filament wire, filaments inside a normal conducting matrix. One current constraint. $I = 0.8I_c \sin(2\pi ft)$
- **Case 2**: 2D: Four-conductor system, four current constraints + applied AC field. $I_i = 0.8I_c \sin(2\pi ft)$, $B_{\text{ext}} = 0.1 \sin(2\pi ft)$ T
- **Case 3**: 3D: Superconducting cylinder, one current constraint. $I = 0.5I_c \sin(2\pi ft)$
Benchmark results

Figure: Case 1: The mesh and the support of the employed cohomology basis function. Decrease from $H$ to $H - \varphi - \Psi$ in DoFs: 15 %, in CPU time: 25 %.
Figure: **Case 1:** The current penetration profiles for both formulations at the peak of the AC quantities. (a): $H$-formulation. (b): $H-\varphi-\Psi$-formulation.
**Benchmark results**

**Figure:** **Case 2:** The mesh and the support of the employed cohomology basis functions. Decrease from \( H \) to \( H-\varphi-\Psi \) in **DoFs:** 34 %, in **CPU time:** 49 %. 

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Figure: Case 2: The current penetration profiles at the peak of the AC quantities. (a) $H$-formulation and (b) $H-\phi-\Psi$-formulation.
Benchmark results

Figure: Case 3: The mesh and the support of the employed cohomology basis function. Decrease from $H$ to $H-\varphi-\Psi$ in DoFs: 43 %, in CPU time: 23 %.
Cohomology basis for a Roebel cable mesh

Figure: Roebel cable mesh by F Grilli and V Zermeno
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Conclusions

- In AC-AC cases, utilizing finite air resistivity in the $H$-formulation may result in a situation that the modeller does not intend to simulate.
- This can be fixed by setting conductivity to zero in the air regions and utilizing cohomology basis functions in the approximation of $H$.
- The $H$-formulation with cohomology basis functions results in faster computations than the traditional one, when the same mesh is utilized.
Thank you