Critical currents and electric fields of individual turns of HTS coils

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Goal & Motivation

… to study the distribution of the magnetic field in the winding of cylindrical HTS coils (both magnitude and orientation)

… to evaluate & analyze the critical currents, electric fields and voltage of individual turns – taking into account the real Ic(B,α) characteristic of the conductor / wire / tape

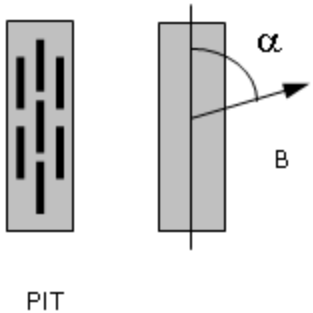
… to evaluate the V(I) characteristic of the coil

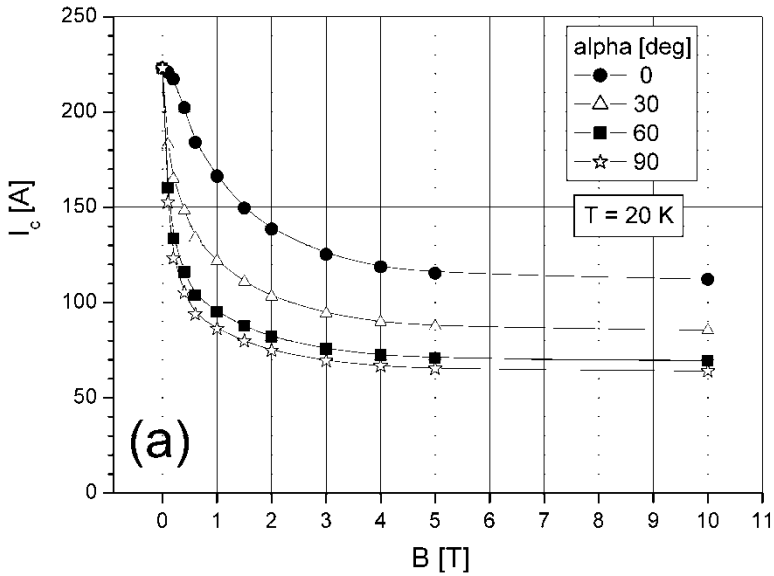
… to determine the factors limiting the critical current of the cylindrical HTS coils

This all – under condition that the HTS coil is subject to external magnetic field parallel with the coils axis (applied to high field insert coils)

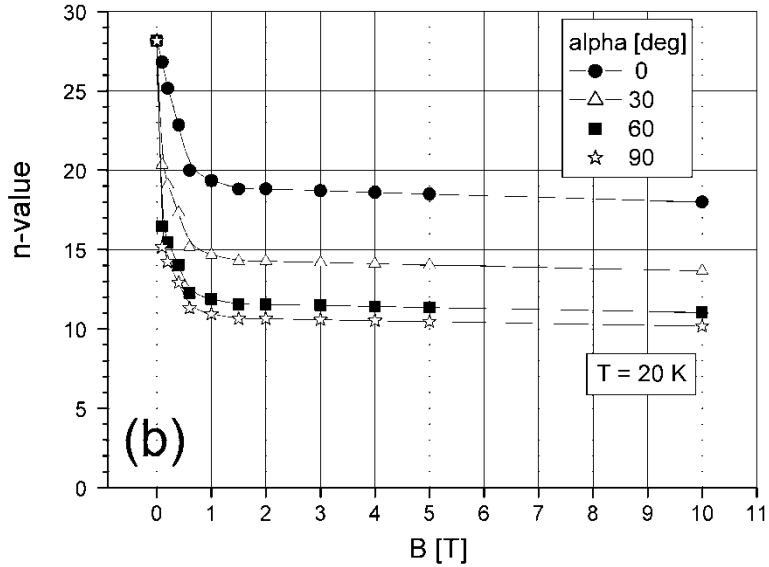
… to pass a message to magnet designers & feedback to conductors producers …

Ic(B)



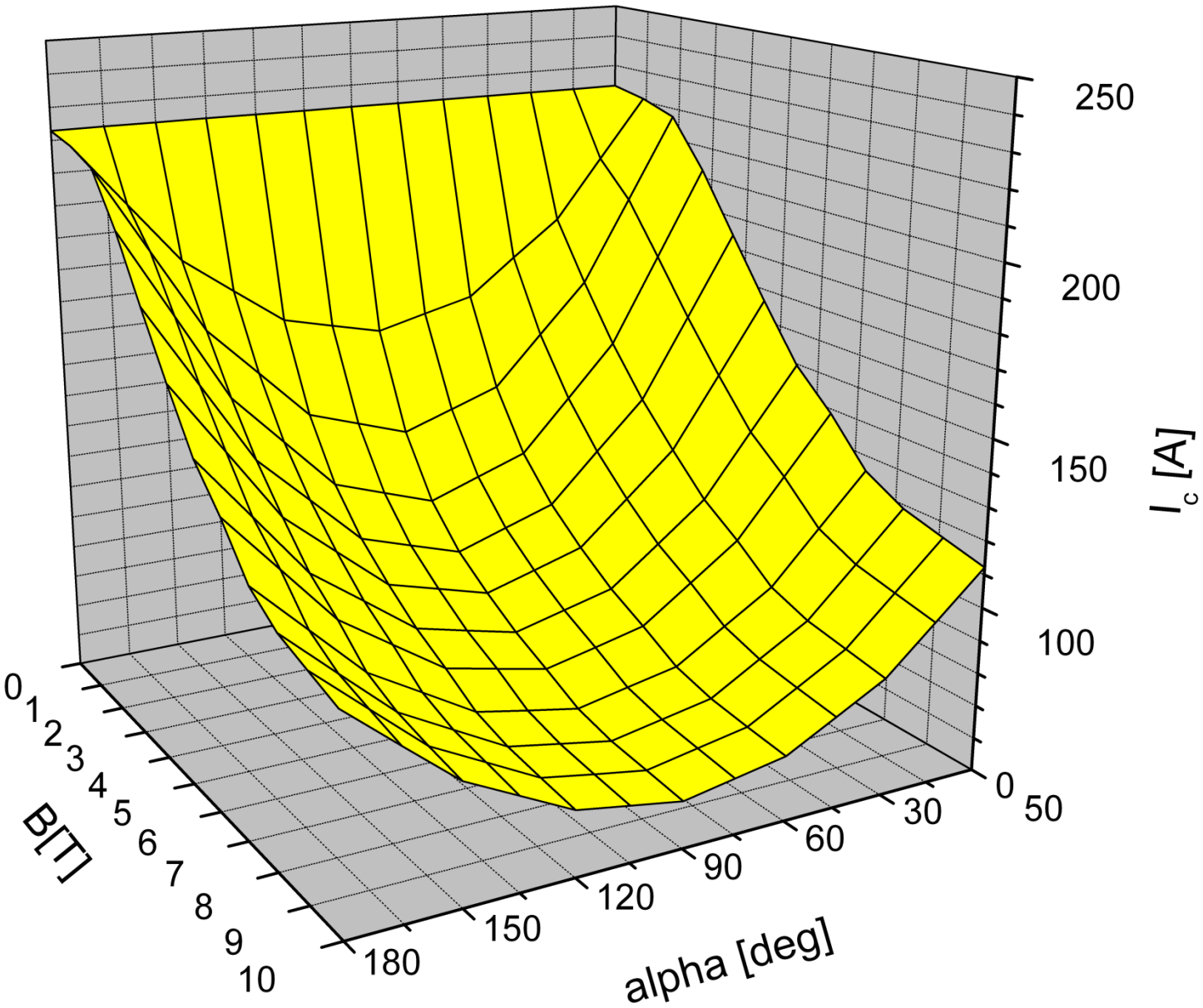


n(B)

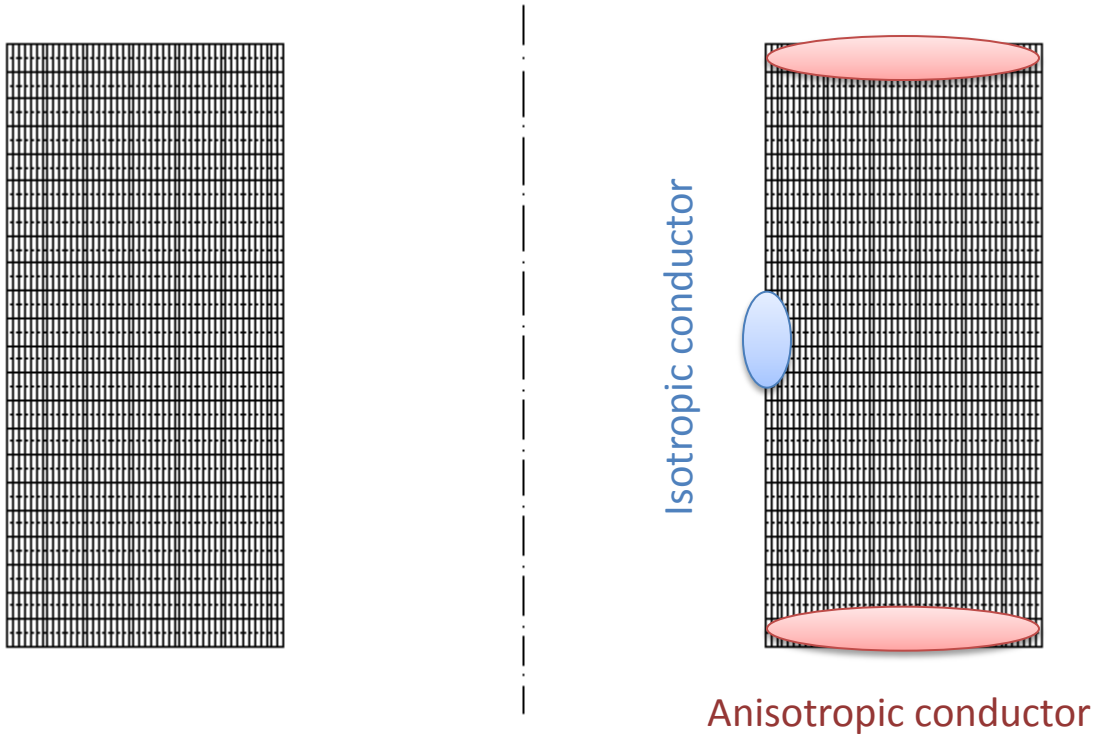


Tape : Bi(2223)/Ag
2.6 x 0.18 mm
85 filaments
(AmSup)

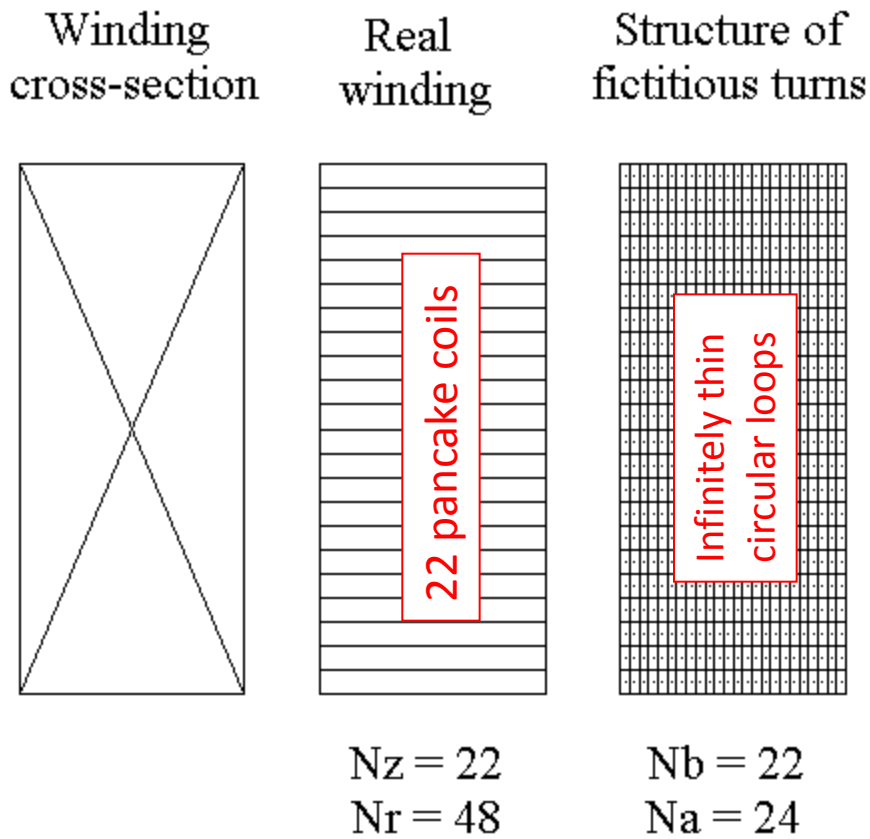
<u>Criterion</u> :
Ec = 1 µV/cm


250
200
150
100
0
I_c [A]
0
1
2
3
4
5
6
7
8
9
10
B[T]
180
150
120
90
60
30
0
alpha [deg]

Cylindrical coil



Model to calculate the magnetic field in the winding of cylindrical coils



No FEM used, but a specific model + software developed !

- *L.Cesnak, D.Kabat : Vypocet magnetickeho pola valcovych cievok s nerovnakou prudovou hustotou, Elektrotechnicky obzor 59 (1970) 338*
- *W.R.Smythe : Static and Dynamic Electricity, Mc Graw-Hill, B.C.N.Y. (1950) 262*
- *M.N.Wilson : Superconducting Magnets, Clarendon Press, Oxford (1983) 45-51*
- *H.E.Knoepfel : Magnetic Fields, J.Wiley & Sons Inc. (2000) 97-104*
- *… others*

Example *: 1 infinitely thin circular loop represents 2 real turns*

In principle, each and every turn can be replaced by one infinitely thin circular loop which is located in the turn center.

Mathematical model

Cylindrical coil – 2D problem

Magnetic field - flux density components : *Br – radial , Bz - axial*

$$B_r(r,z) = I_{op} G_r = I_{op} \frac{N}{N_a N_b} \frac{\mu_o}{2\pi} \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \frac{z - b_{ij}}{r} f_1 \left[f_2 E(k_{ij}) - K(k_{ij}) \right]$$

$$B_z(r,z) = I_{op} G_z = I_{op} \frac{N}{N_a N_b} \frac{\mu_o}{2\pi} \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} f_1 \left[f_3 E(k_{ij}) + K(k_{ij}) \right]$$

N – number of real turns

Na*Nb – number of fictitious turns

Iop – operating current

aij , bij – coordinates of the fictitious turn

$$f_1 = \frac{1}{\sqrt{(r + a_{ij})^2 + (z - b_{ij})^2}}$$

$$f_2 = \frac{r^2 + a_{ij}^2 + (z - b_{ij})^2}{(r - a_{ij})^2 + (z - b_{ij})^2}$$

$$f_3 = \frac{-r^2 + a_{ij}^2 - (z - b_{ij})^2}{(r - a_{ij})^2 + (z - b_{ij})^2}$$

$K(k_{ij})$ $E(k_{ij})$

$$k_{ij} = \sqrt{\frac{4ra_{ij}}{(r + a_{ij})^2 + (z - b_{ij})^2}}$$

Complete elliptic integrals of the 1st and 2nd kind with the modulus kij

Critical current , Electric field , Voltage

- $I_c = I_c(B,\alpha)$ & $n = n(B,\alpha)$ **both are linearized in parts !** **Tape**

- $B_{r,z\,ij} = I_{op}\, G_{r,z\,ij} + B_{r,z\,ext}$ $\alpha_{ij} = \tan^{-1}\left(\frac{I_{op}G_{rij} + B_{rext}}{I_{op}G_{zij} + B_{zext}}\right)$ **Turn in the winding**

Iop – operating current

$$I_{cij} = I_1 + D_I\left[\sqrt{(I_{op}G_{rij} + B_{rext})^2 + (I_{op}G_{zij} + B_{zext})^2} - B_1\right]$$

$D_I = (I_2 - I_1)/(B_2 - B_1)$

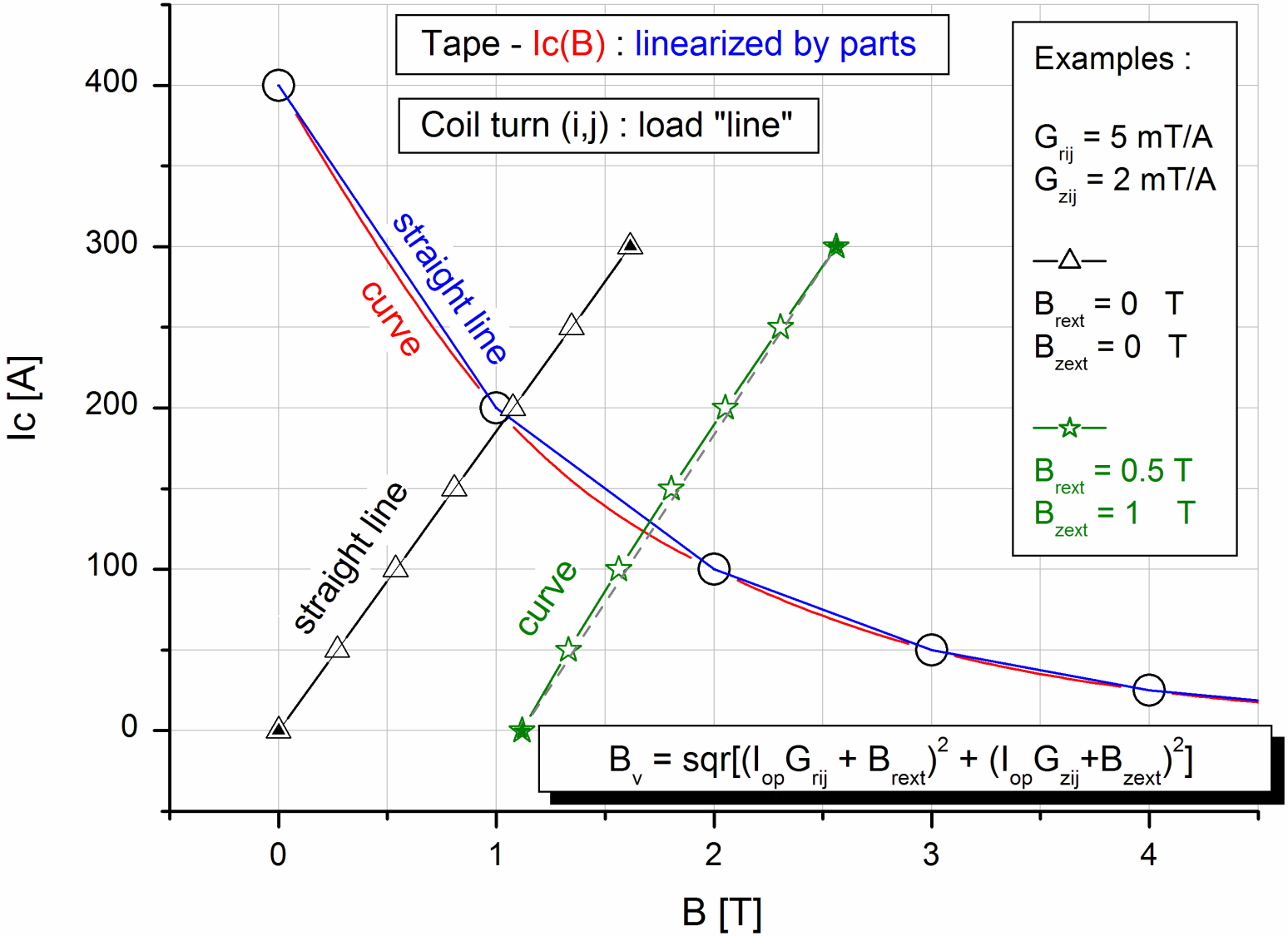
Field magnitude Bvij

$$n_{ij} = n_1 + D_n\left[\sqrt{(I_{op}G_{rij} + B_{rext})^2 + (I_{op}G_{zij} + B_{zext})^2} - B_1\right]$$

$D_n = (n_2 - n_1)/(B_2 - B_1)$

Power law $E_{ij} = E_c\left(\frac{I_{op}}{I_c(B_{vij},\alpha_{ij})}\right)^{n_{ij}(B_{vij},\alpha_{ij})}$ $V_{ij} = 2\pi \cdot r_{ij} \cdot E_{ij}$

- $V_j = \sum_i V_{ij}$ $V = \sum_{i,j} V_{ij}$ **pancake , coil**


Tape - Ic(B) : linearized by parts
Coil turn (i,j) : load "line"
Examples :
G_{rij} = 5 mT/A
G_{zij} = 2 mT/A
B_{rext} = 0 T
B_{zext} = 0 T
B_{rext} = 0.5 T
B_{zext} = 1 T
straight line
curve
straight line
curve
$B_v = sqr[(I_{op}G_{rij} + B_{rext})^2 + (I_{op}G_{zij} + B_{zext})^2]$
Ic [A]
B [T]
400
300
200
100
0
0
1
2
3
4

(A) <u>Specific case</u> : Critical current of the turn in coil's self-field (i.e. Bext = 0)

Critical current of the turn (i,j) : Iop = Icij

The set of 2 linear equations :

1) Coil : Turn (i,j) – field magnitude

$$B_{vij} = I_{cij} \cdot G_{vij} = I_{cij}\sqrt{G_{rij}^2 + G_{zij}^2}$$

$$\alpha_{ij} = \tan^{-1}\left(\frac{B_{rij}}{B_{zij}}\right) = \tan^{-1}\left(\frac{I_{cij}G_{rij}}{I_{cij}G_{zij}}\right) = \tan^{-1}\left(\frac{G_{rij}}{G_{zij}}\right)$$ **- angle**

2) Tape : Ic(B,alpha) linear in parts

$$I_{cij} - I_n = D_n \left(B_{vij} - B_n\right)$$

$D_n = (I_{n+1} - I_n)/(B_{n+1} - B_n)$

the slope between the two points of the $I_c(B)$ characteristic which corresponds to angle α_{ij}

$$I_{cij} = \frac{I_n - D_n B_n}{1 - D_n G_{vij}}$$

(B) <u>General case</u> : Critical current of the turn in coil's self-field + external field (Brext ; Bzext)

Critical current of the turn (i,j) : Iop = Icij

The set of 2 equations :

1) Coil : Turn (i,j) – field magnitude

$$B_{r,z\,ij} = I_{cij}\,G_{r,z\,ij} + B_{r,z\,ext}$$

$$\alpha_{ij} = \tan^{-1}\left(\frac{I_{cij}G_{rij} + B_{rext}}{I_{cij}G_{zij} + B_{zext}}\right)$$ **- angle**

$$B_{vij} = \sqrt{(I_{cij}G_{rij} + B_{rext})^2 + (I_{cij}G_{zij} + B_{zext})^2}$$

2) Tape : Ic(B,alpha) linear in parts

$$I_{cij} - I_n = D_n\,(B_{vij} - B_n)$$

where $D_n = (I_{n+1} - I_n)/(B_{n+1} - B_n)$

Final non-linear (quadratic) equation with respect to Icij

$$I_{cij} - I_n = D_n\left(\sqrt{(I_{cij}G_{rij} + B_{rext})^2 + (I_{cij}G_{zij} + B_{zext})^2} - B_n\right)$$

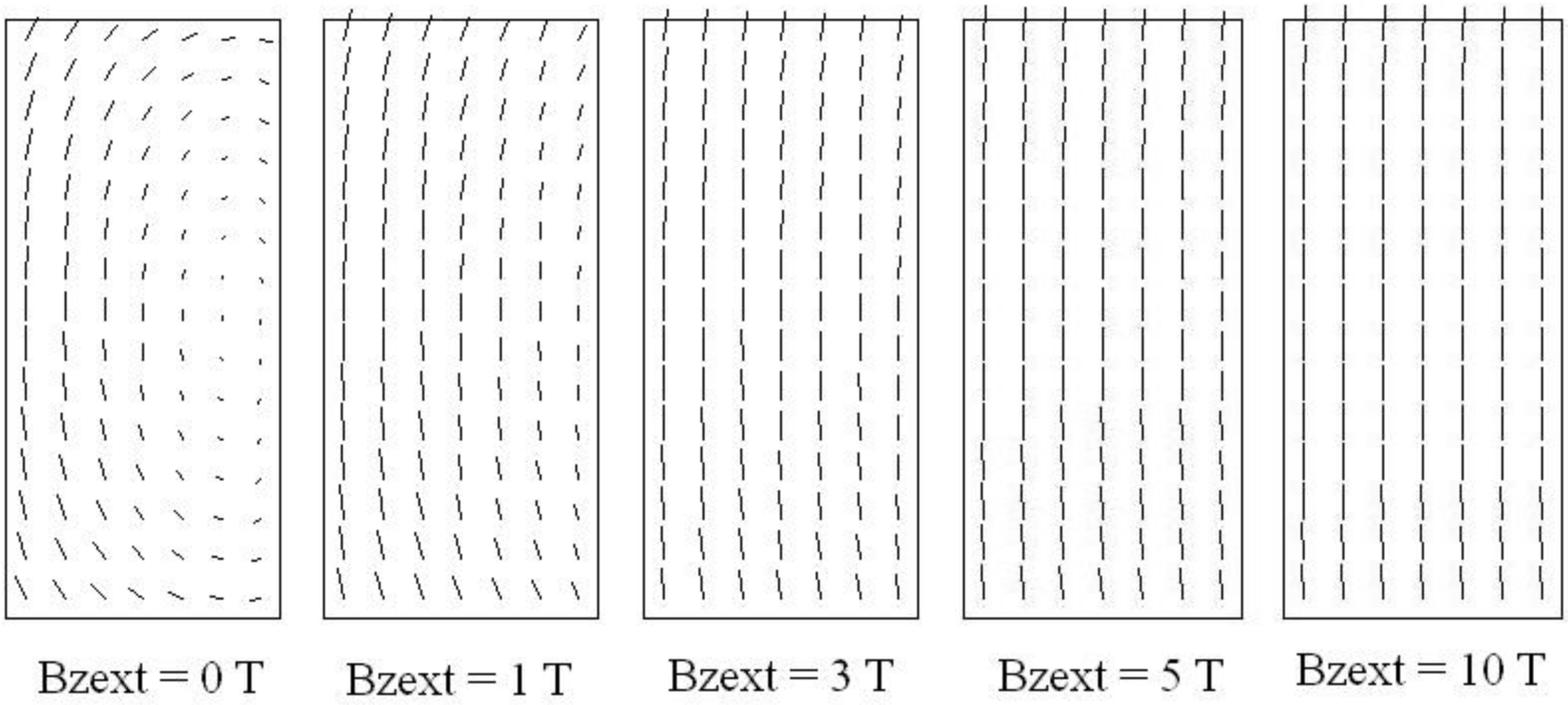
The most simple way to solve : <u>iterative procedure</u> - increase of Iop in small steps δIop

Model coil specification

• Inner winding radius	a1 [mm]	27.5		
• Outer winding radius	a2 [mm]	59		
• Coil length	2b [mm]	69		
• Number of pancakes	Nz	22	(Nb = 22)	**Fictitious winding**
• Number of turns per pancake	Nr	48	(Na = 24)	
• Overall turns number	N	1056		
	Sefr [mm*2]	2.058		
• Tape length (overall)	l [m]	287		
• Central field	k0 [mT/A]	12.123		
• Self inductance	L [mH]	54.9		
• **Critical current @ 20 K**	**Icmin [A]**	**101.1 A**		

Distribution of the magnetic field in the coil winding @ different Bzext

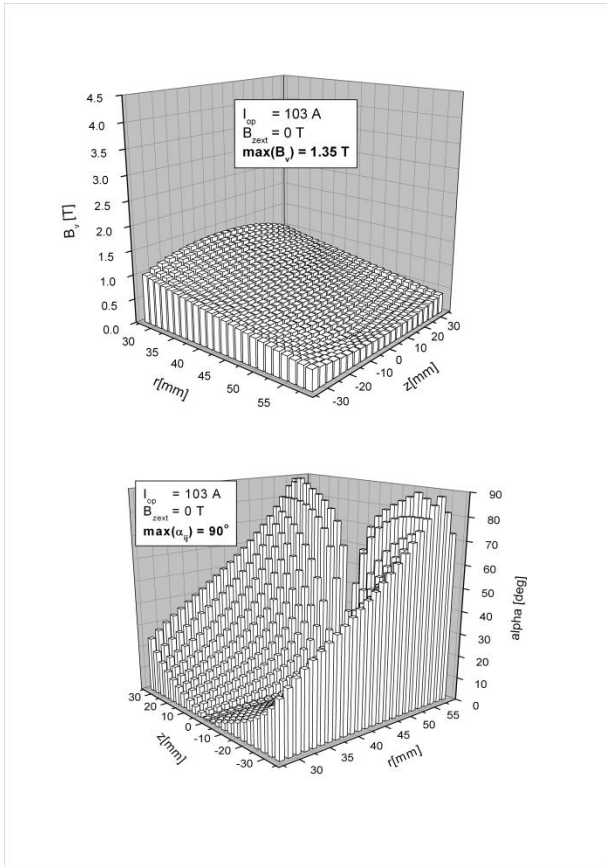
Iop = 103 A ; Icmin = min(Icij) = 101.1 A ; i.e. Iop > Icmin

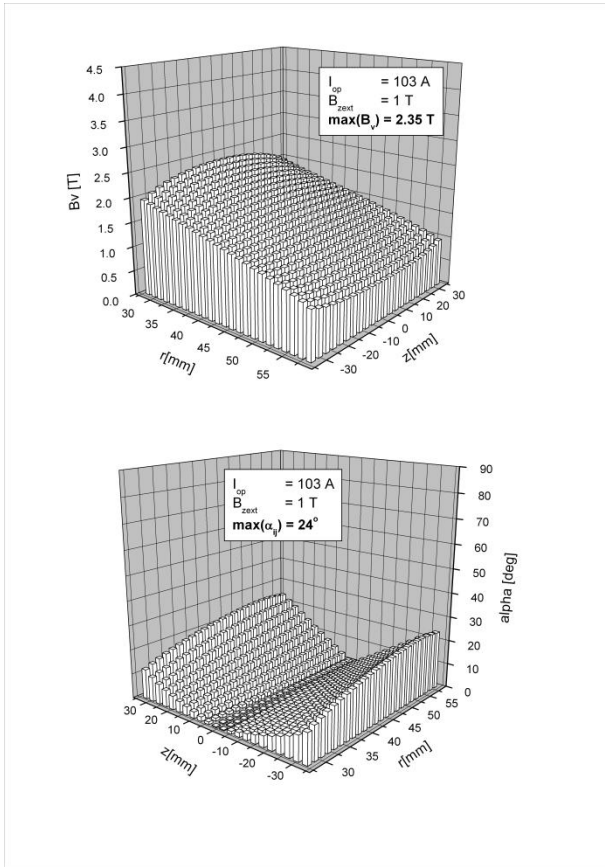


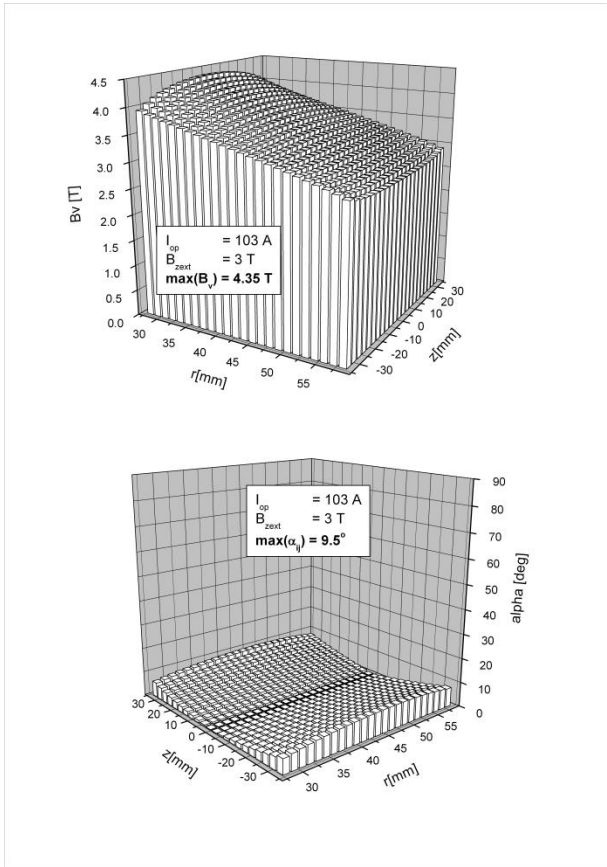
Bv	**[T]**	**0 - 1.35 T**	**0.76 - 2.35**	**2.75 - 4.35**	**9.76 - 11.35**
max (α)	**[deg]**	**90**	**24**	**9.5**	**3.1**
max (Br)	**[T]**	**0.56**	**0.56**	**0.56**	**0.56**

The length of the magnetic flux density vectors is normalized to respective maximum value of the field

Magnetic flux density in the winding – absolute value Bv(i,j)

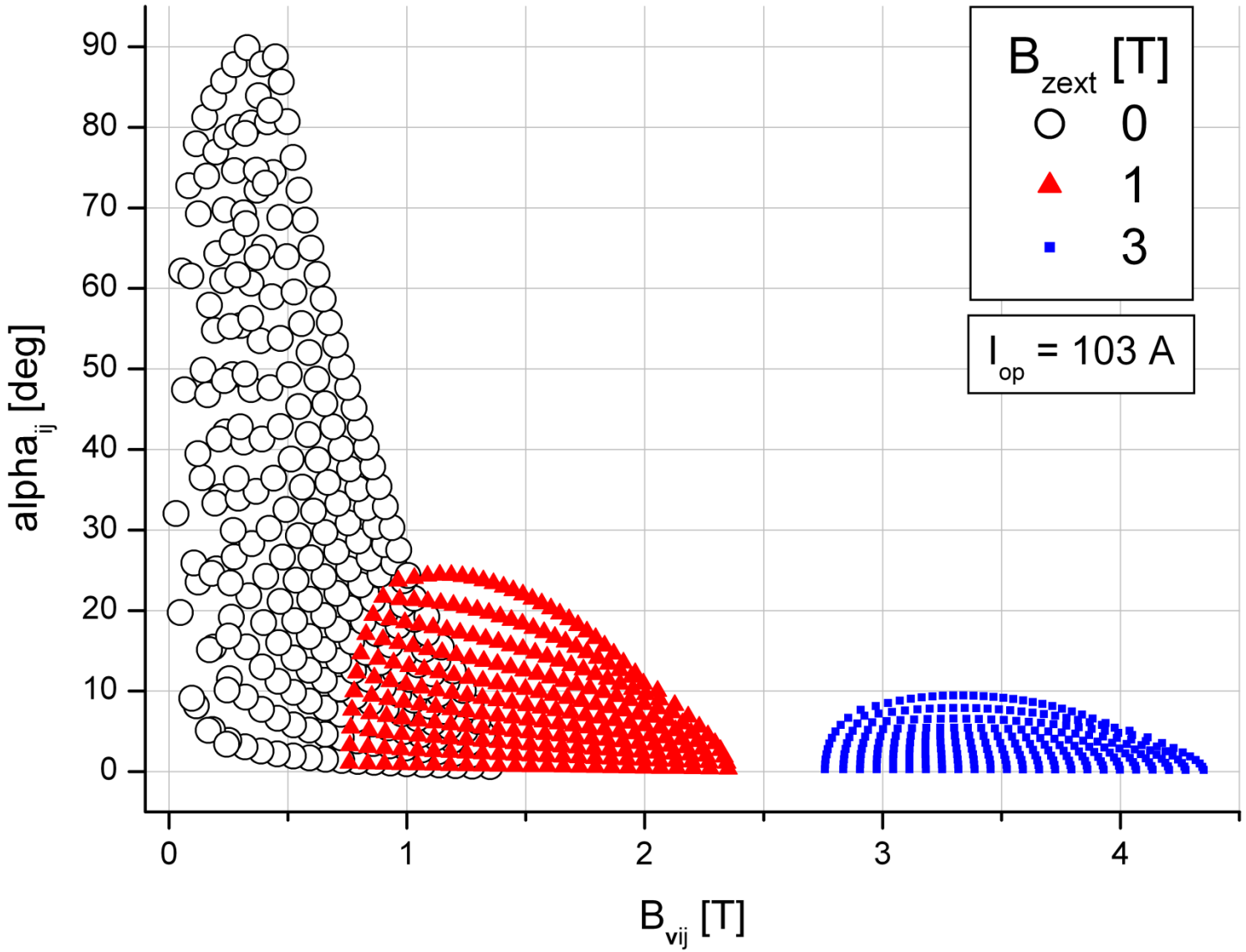




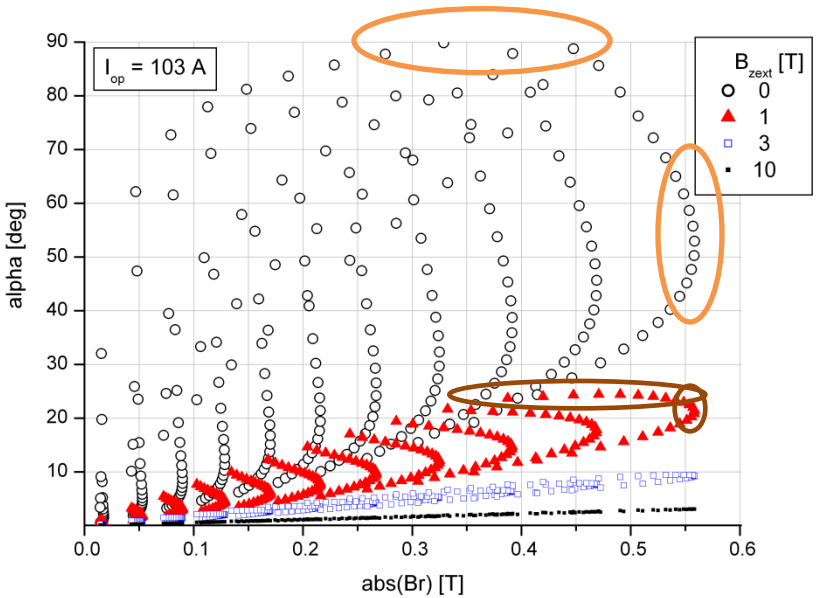


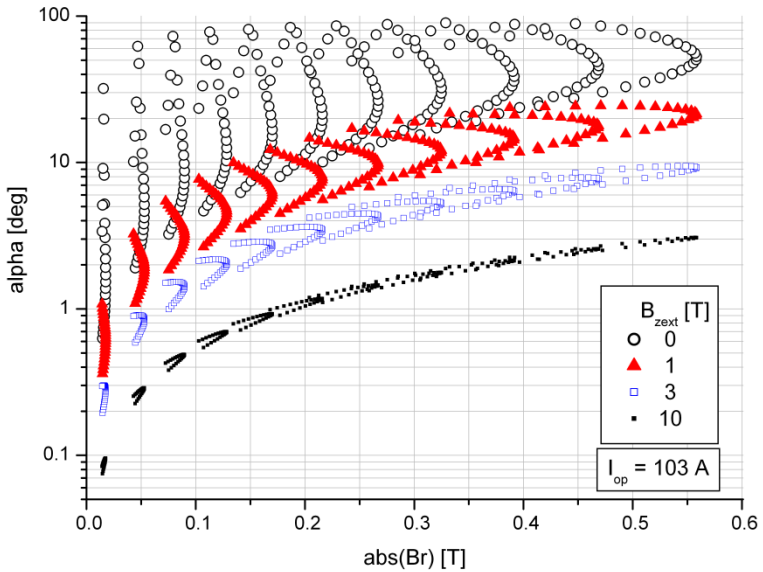
Magnetic field orientation – angle α(i,j)

Winding : Field orientation vs field magnitude



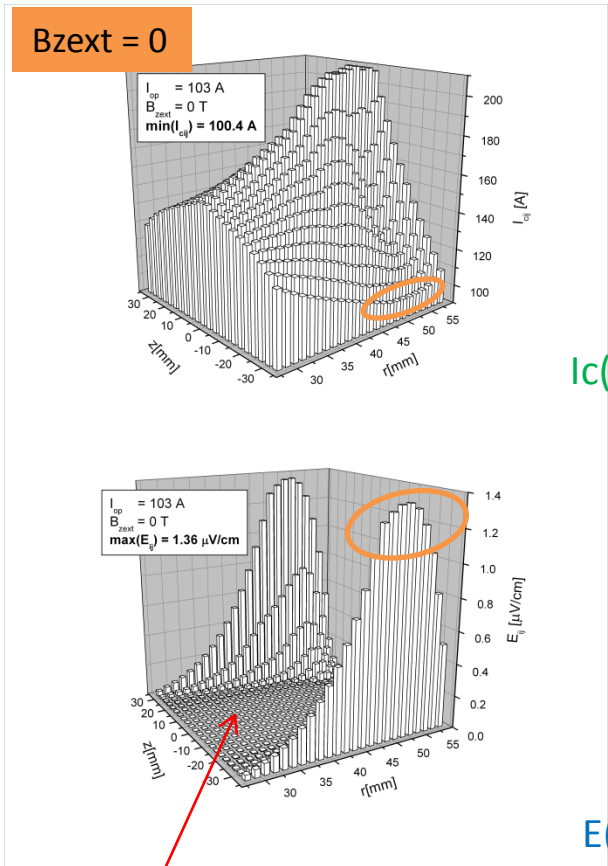
Winding : Correlation between the field orientation (α) and abs(Br)

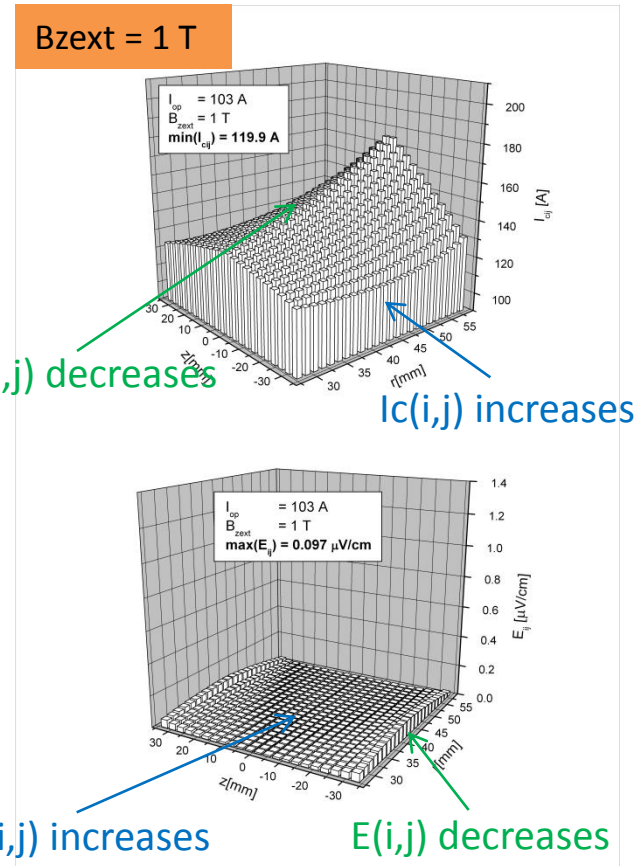


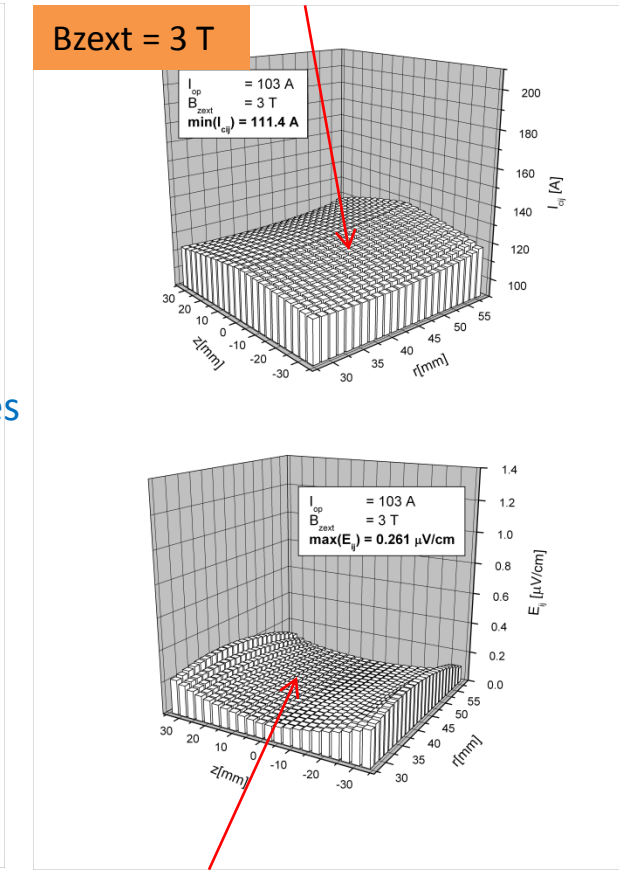


Critical currents Ic(i,j) of individual turns

Only small differences in Ic(I,j)





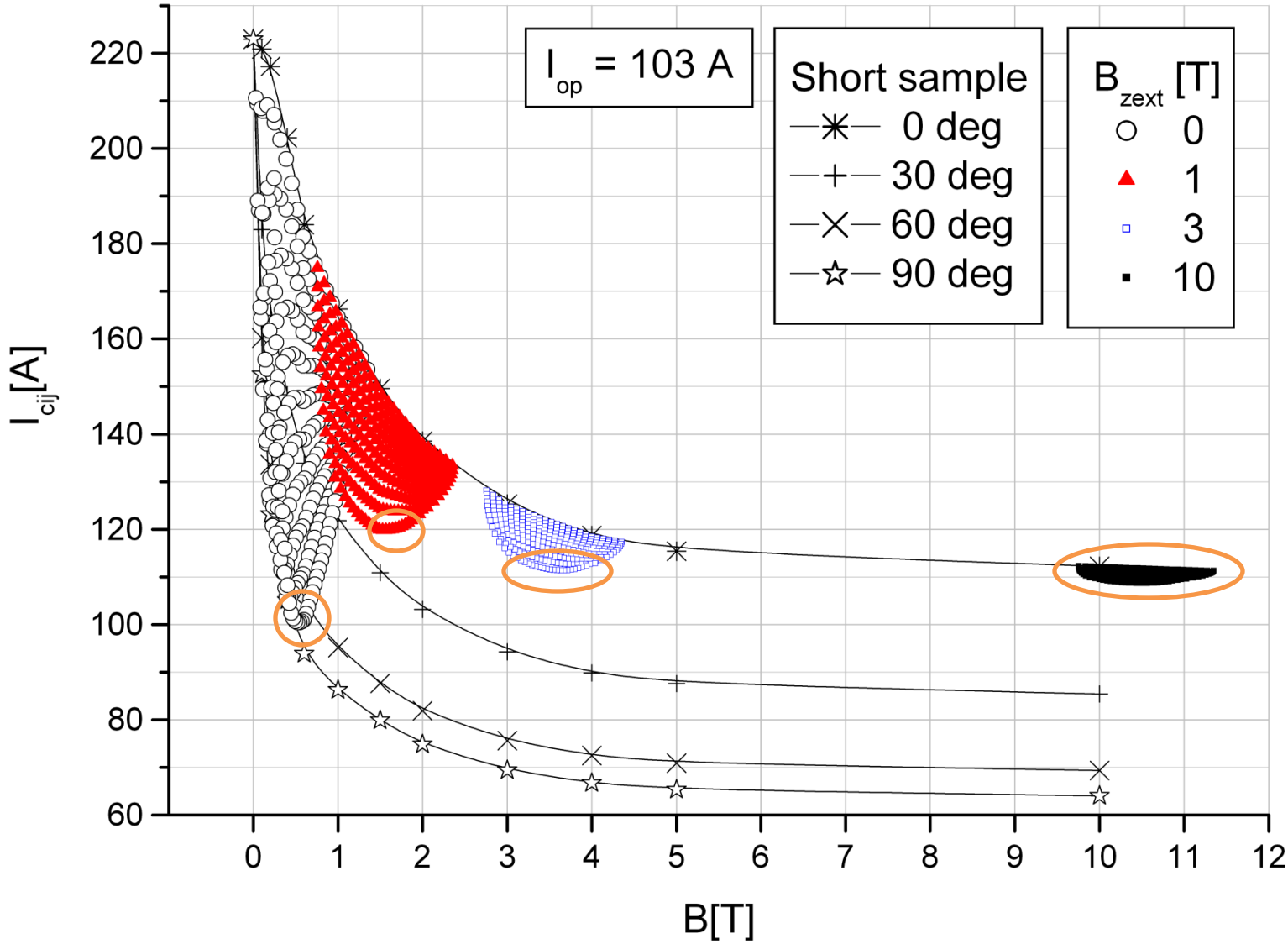


No electric field

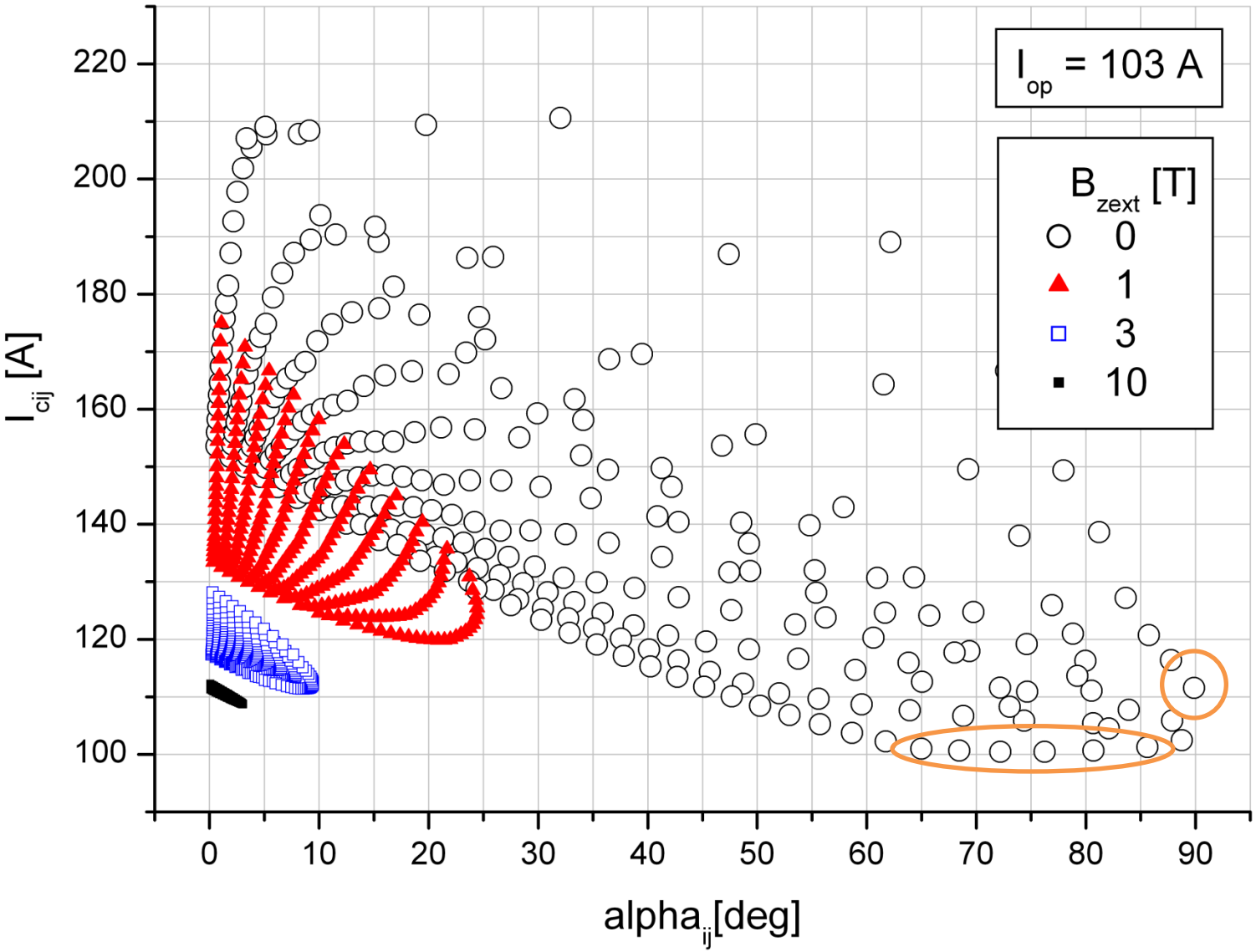
Electric field E(i,j)

Electric field area spreads over the whole winding

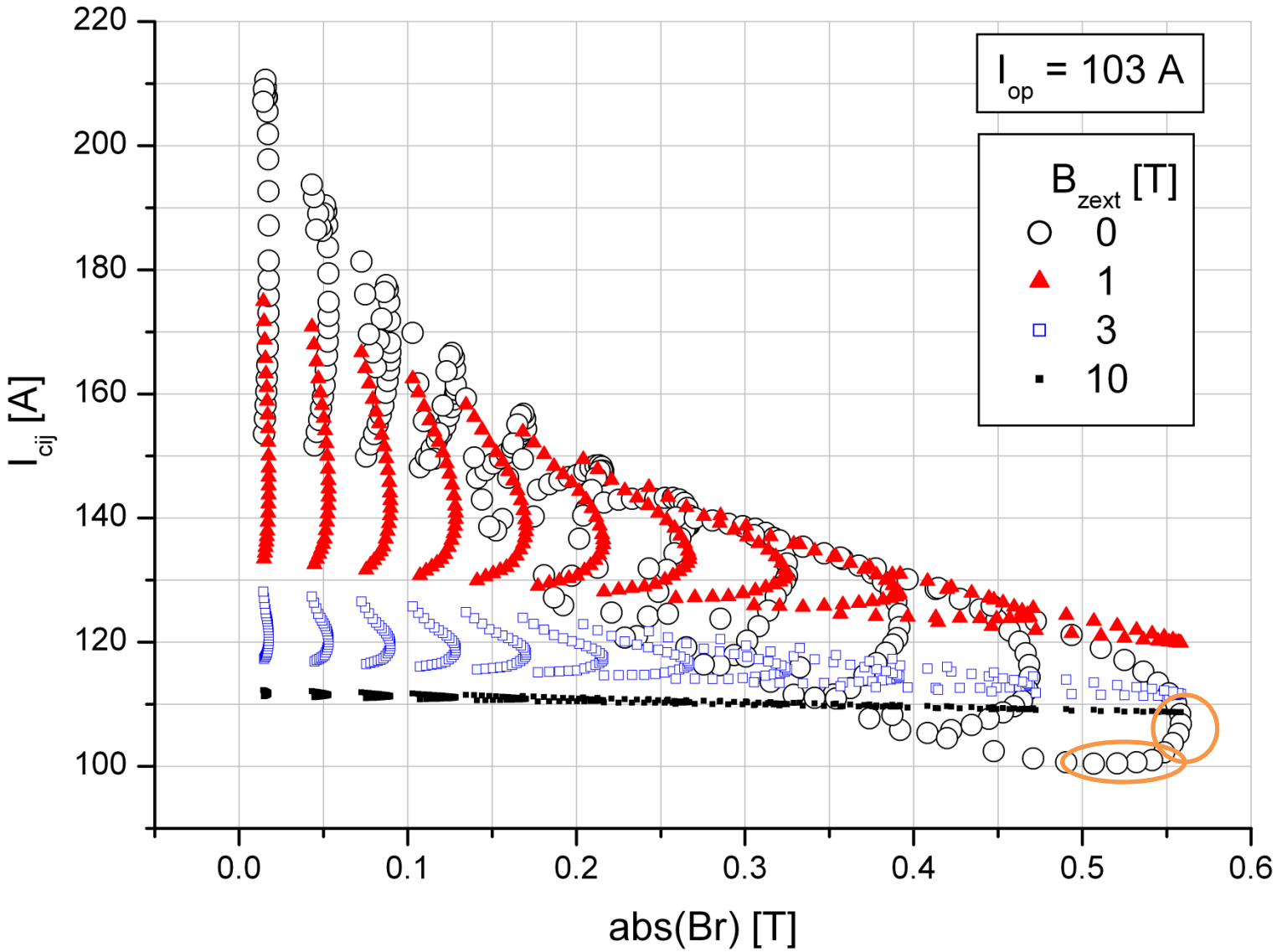
Critical current Ic(i,j) vs magnetic field magnitude Bv(i,j)



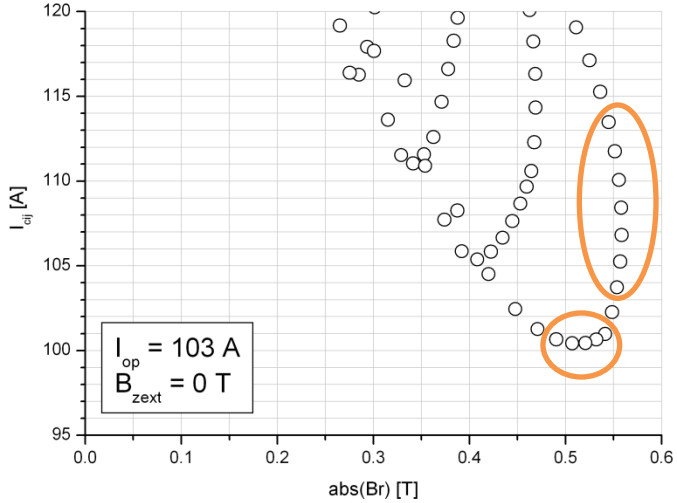
Critical current Ic(i,j) vs magnetic field orientation α(i,j)

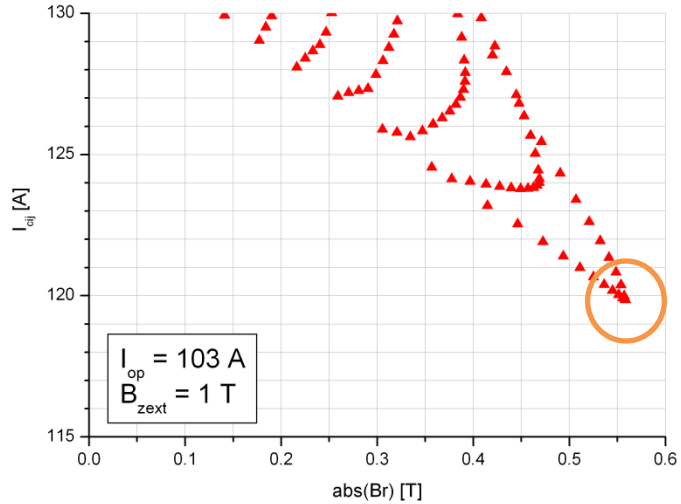


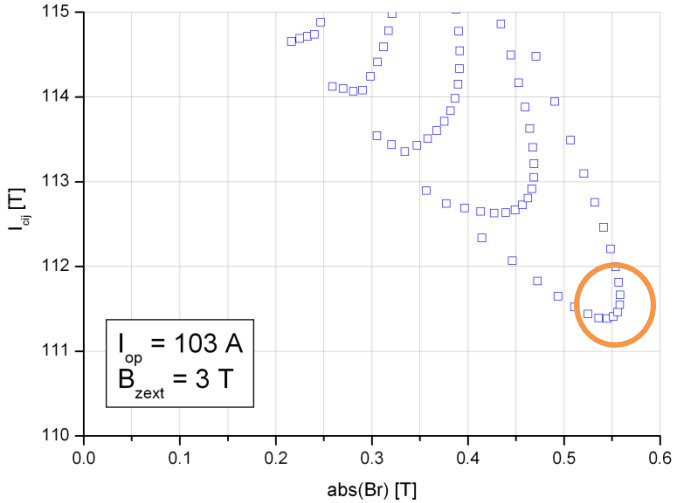
Critical current Ic(i,j) vs radial component of the magnetic field Br(i,j)

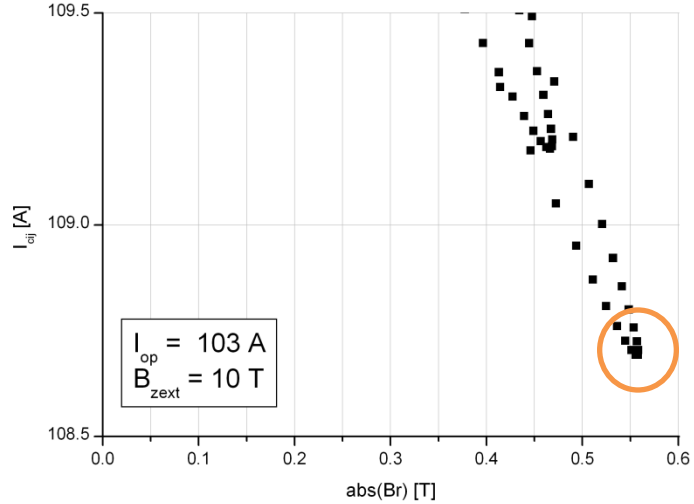


Critical current Ic(i,j) vs radial magnetic field Br(i,j) - detail

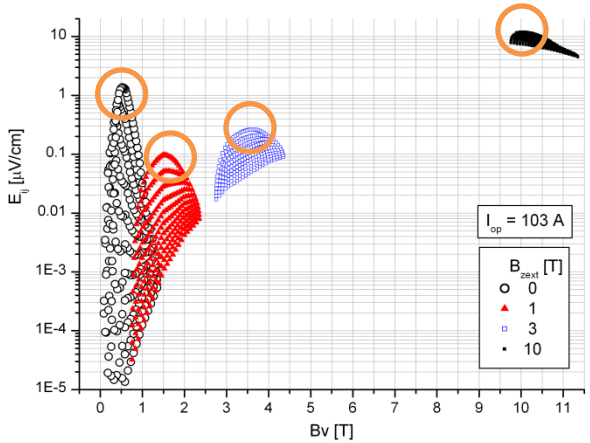




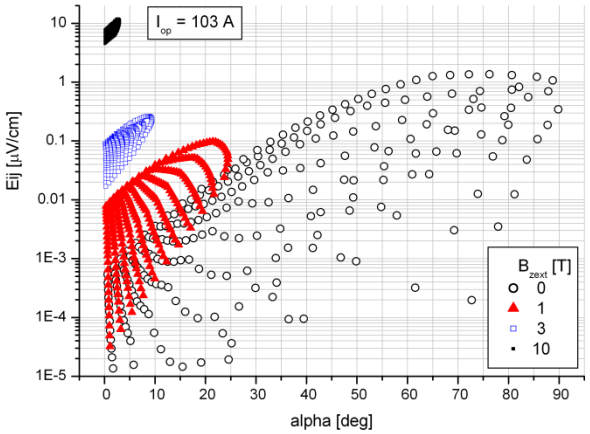




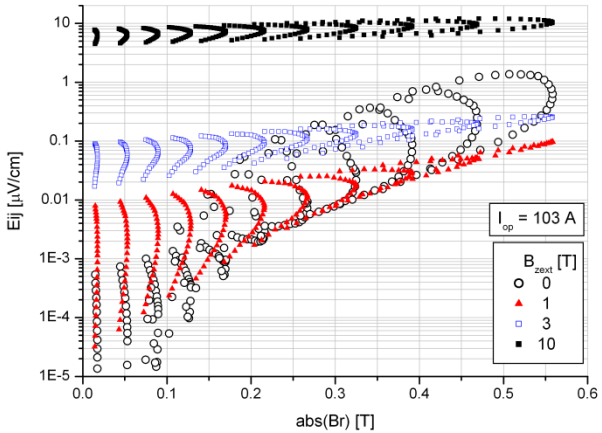
Eij vs Bv



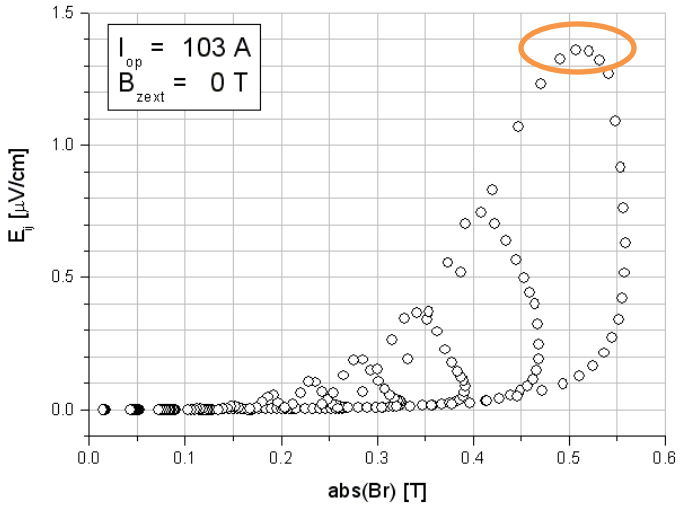
Eij vs α

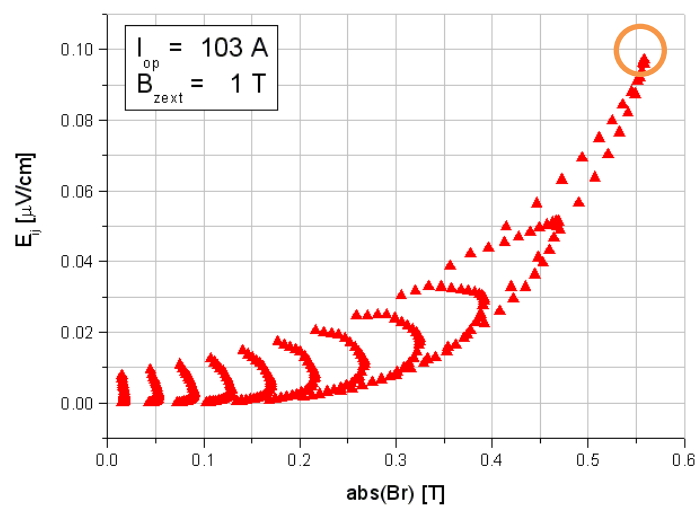


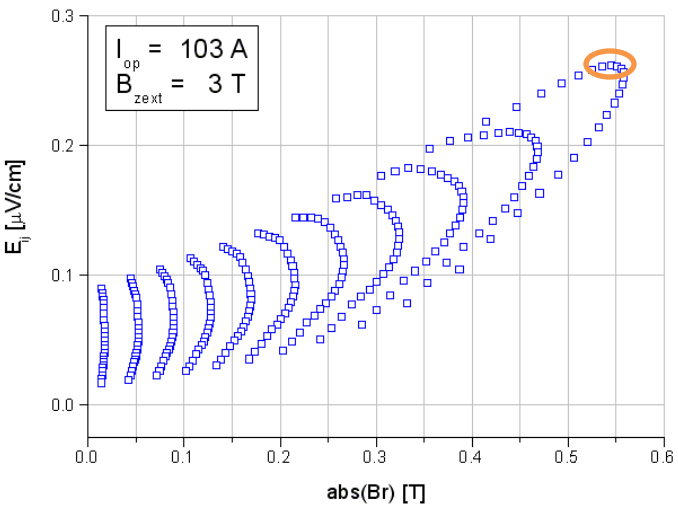
Eij vs abs(Br)

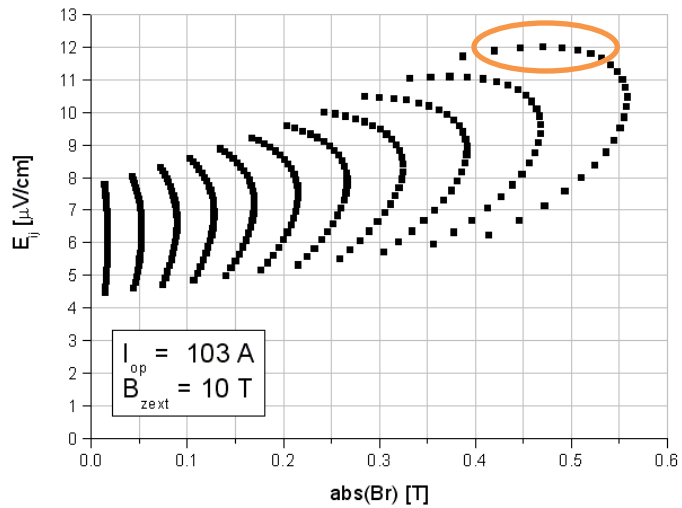


Electric field E(i,j) vs radial magnetic field Br(i,j) - detail

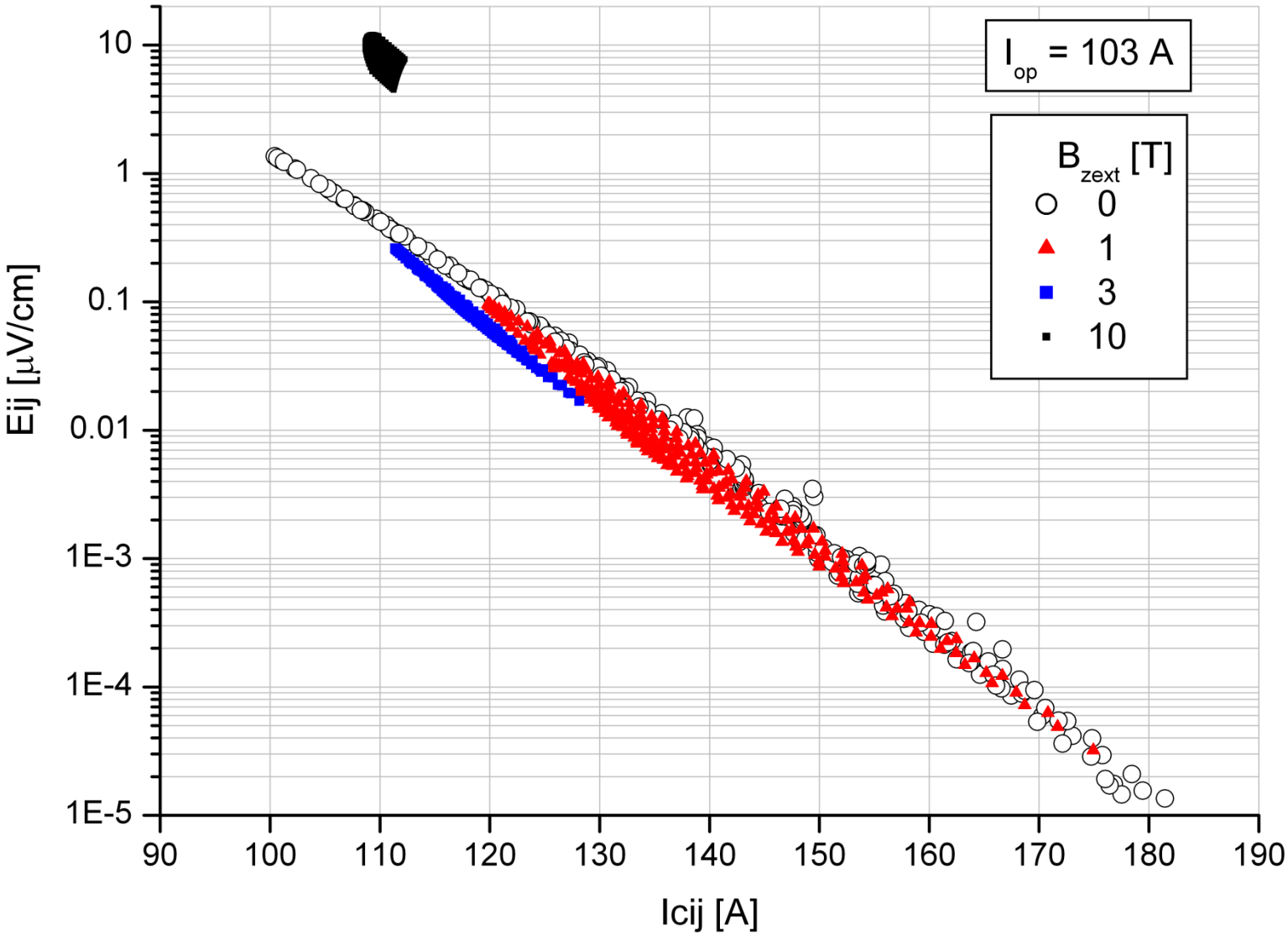




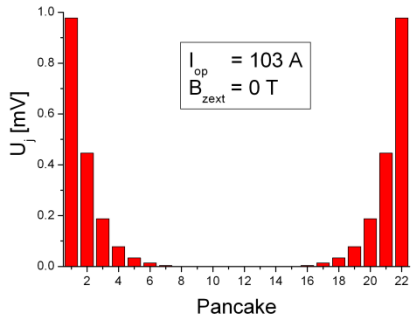


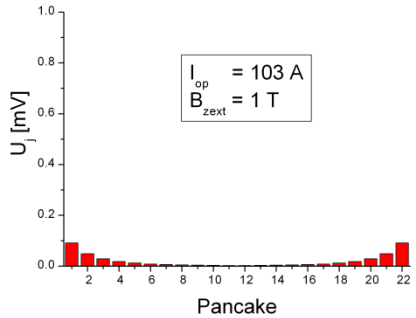


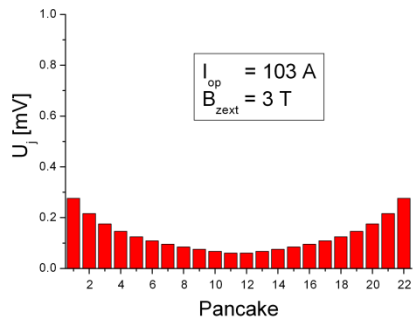
Individual turns - Correlation between electric field E(i,j) and critical current Ic(i,j)

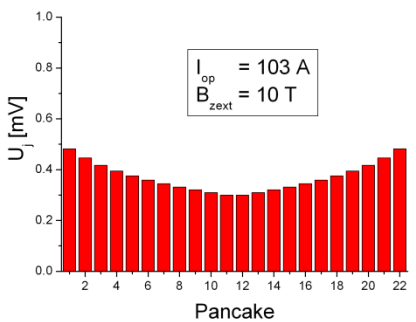


Voltage on individual pancakes
parameter : Bzext

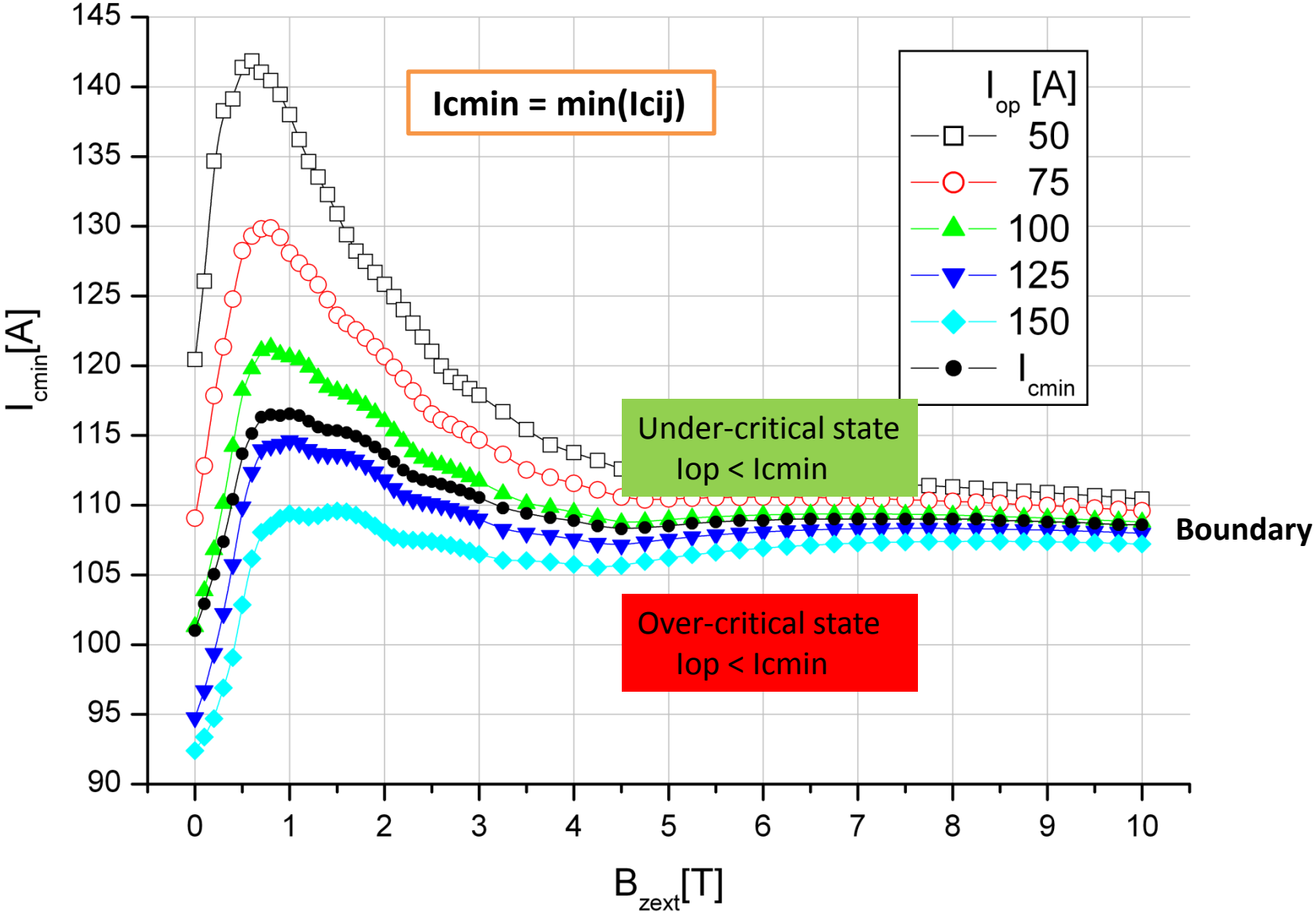






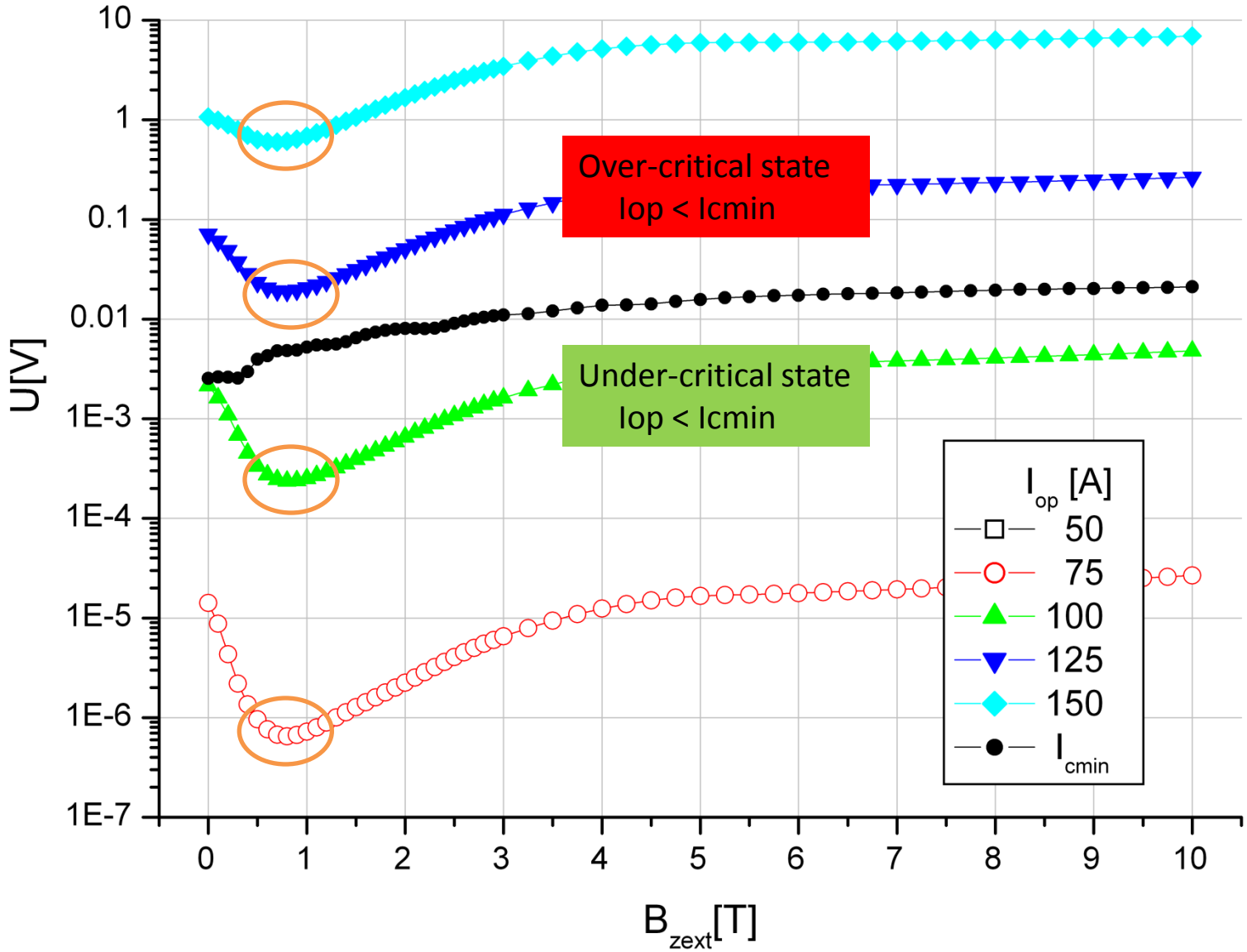


Critical current of the coil **Icmin** @ different operating currents **Iop**

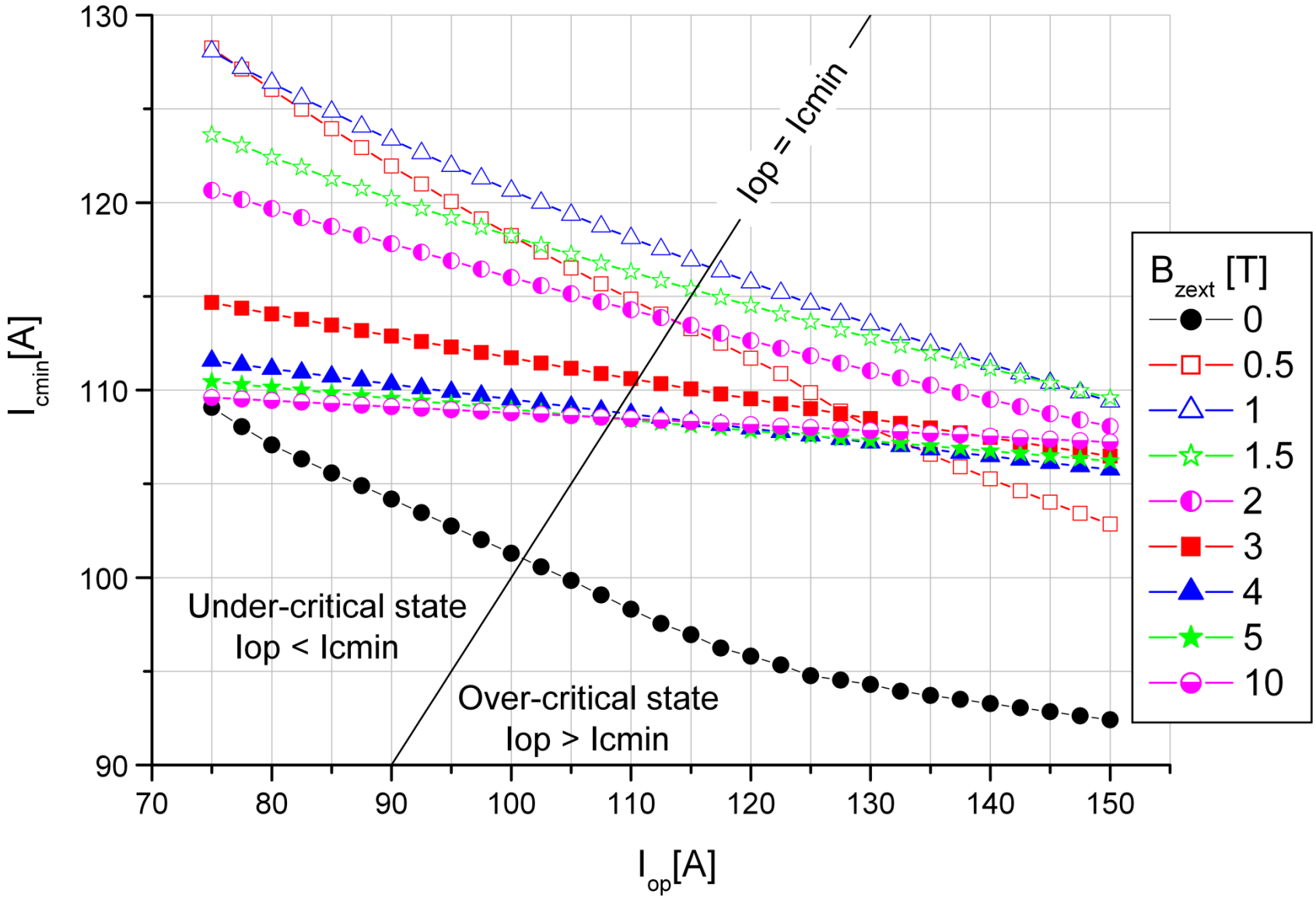


Voltage on coil terminals **U** as a function of **Bzext**

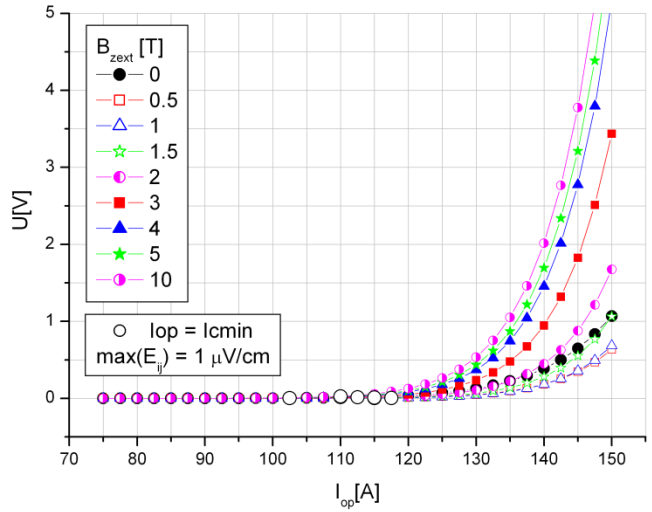
Parameter : operating current ***Iop***

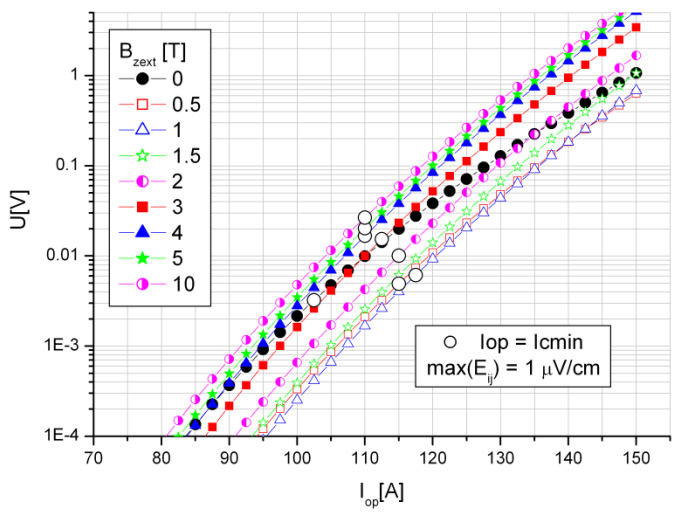


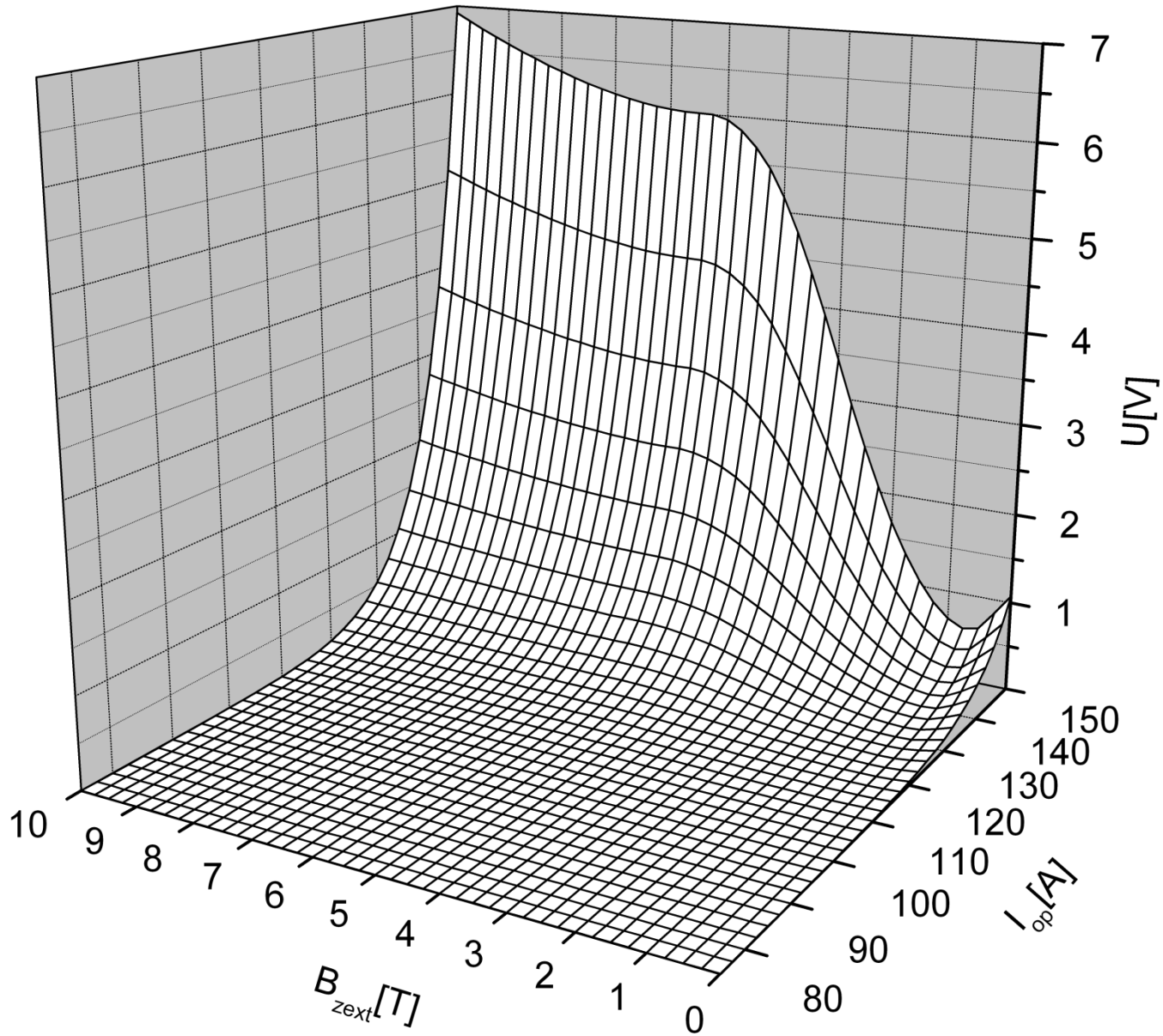
Critical current of the coils **Icmin** vs operating current **Iop**



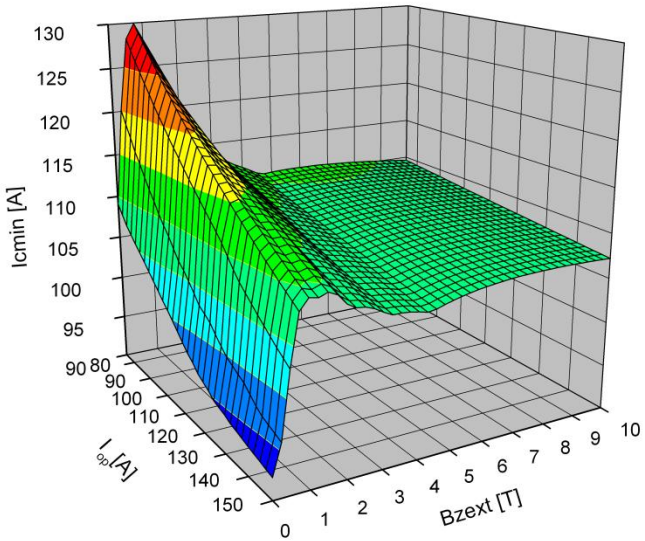
V(I) characteristic of the coil
Parameter : Bzext



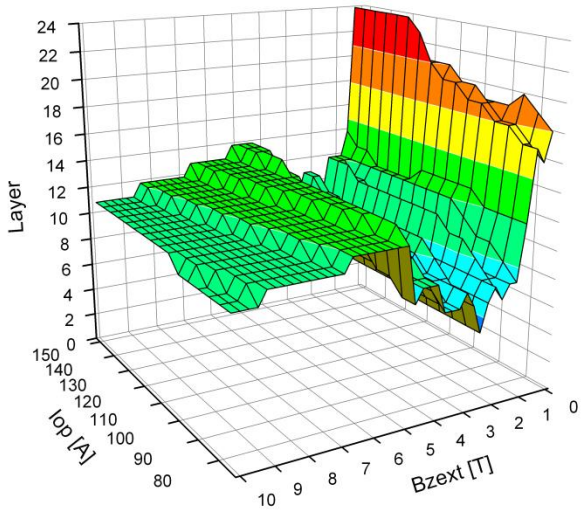



7
6
5
4
3
2
1
U[V]
150
140
130
120
110
100
90
80
I_op[A]
10
9
8
7
6
5
4
3
2
1
0
B_zext[T]

Icmin = f(Iop , Bzext)



Position of the weak turn in the winding



CONCLUSIONS

Factors limiting both :

- the critical current of the individual turn Icij
- the critical current Icmin = min(Icij) of the HTS coil :

- Anisotropy in the Ic(B) of the tape ; magnetic field amplitude + field orientation
- Icij – **is not determined exclusively by the radial component of the magnetic field.**
  It is a compromise between the field magnitude and the field orientation.

Weak turns of the HTS cylindrical coil with uniform winding structure are always located in the outermost pancakes. Their accurate position in the pancakes depends on the winding geometry and external field. In case of high field insert coils it is the mid-layer.

Numerical simulations predict **rather unexpected behaviour of the HTS coils** if an external magnetic field Bzext parallel with the coil axis is applied :

- High magnetic loading (field magnitude) of weak turns is compensated by the decrease of an angle between the tape surface and field orientation.

Consequence :

- **increase in the critical current Icmin** @ low external fields Bzext
- **decrease of the voltage U** on magnet terminals

- Icij-set moves into the area of Ic(B,α) characteristic which corresponds to low angles α if Bzext increases,

i.e.

- **The anisotropy in Ic(B) is not applied / does not play any role at high Bzext values**

Consequence :

In case of the HTS insert coils it is necessary to concentrate rather on the production of the tapes with the current carrying capacity as high as possible than on the attempt to decrease the anisotropy in Ic(B)

… the experience shows that the decrease of the Ic(B) anisotropy by changing the filament architecture in the tape is always accompanied by a simultaneous decrease of the current carrying capacity …

The above results were published in *:*

- *Supercond. Sci. Technol.* ***15*** *(2002) 1499-1506*
- *Physica C* ***471*** *(2011) 1680-1688*
- *Supercond. Sci. Technol.* ***26*** *(2013) 125002*
- *Supercond. Sci. Technol.* ***12*** *(1999) 62-68*
- *Supercond. Sci. Technol.* ***10*** *(1997) 7-16*