Critical currents and electric fields of individual turns of HTS coils

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Goal & Motivation

... to study the distribution of the magnetic field in the winding of cylindrical HTS coils (both magnitude and orientation)

... to evaluate & analyze the critical currents, electric fields and voltage of individual turns – taking into account the real $I_c(B,\alpha)$ characteristic of the conductor / wire / tape

... to evaluate the $V(I)$ characteristic of the coil

... to determine the factors limiting the critical current of the cylindrical HTS coils

This all – under condition that the HTS coil is subject to external magnetic field parallel with the coils axis (applied to high field insert coils)

... to pass a message to magnet designers & feedback to conductors producers ...
Tape: Bi(2223)/Ag
2.6 x 0.18 mm
85 filaments
(AmSup)

Criterion:
Ec = 1 μV/cm
Cylindrical coil

Isotropic conductor

Anisotropic conductor
Model to calculate the magnetic field in the winding of cylindrical coils

In principle, each and every turn can be replaced by one infinitely thin circular loop which is located in the turn center.

Example: 1 infinitely thin circular loop represents 2 real turns

No FEM used, but a specific model + software developed!

- L.Cesnak, D.Kabat: Vypocet magnetického pola valcových cievok s nerovnakou prudovou hustotou, Elektrotechnicky obzor 59 (1970) 338
- W.R.Smythe: Static and Dynamic Electricity, Mc Graw-Hill, B.C.N.Y. (1950) 262
- ... others

Winding cross-section  | Real winding  | Structure of fictitious turns
---|---|---
Nz = 22  | 22 pancake coils  | Infinitely thin circular loops
Nr = 48  |  |  
Nb = 22  |  |  
Na = 24  |  |  

- L.Cesnak, D.Kabat: Vypocet magnetického pola valcových cievok s nerovnakou prudovou hustotou, Elektrotechnicky obzor 59 (1970) 338
Mathematical model

Cylindrical coil – 2D problem
Magnetic field - flux density components: \( B_r - radial \), \( B_z - axial \)

\[
B_r(r, z) = I_{op} G_r = I_{op} \frac{N_a}{N_a N_b} \frac{\mu_0}{2\pi} \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \frac{z - b_{ij}}{r} f_1 E(k_{ij}) - K(k_{ij})
\]

\[
B_z(r, z) = I_{op} G_z = I_{op} \frac{N_a}{N_a N_b} \frac{\mu_0}{2\pi} \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} f_1 E(k_{ij}) + K(k_{ij})
\]

\[
f_1 = \frac{1}{\sqrt{(r + a_{ij})^2 + (z - b_{ij})^2}}
\]

\[
f_2 = \frac{r^2 + a_{ij}^2 + (z - b_{ij})^2}{(r - a_{ij})^2 + (z - b_{ij})^2}
\]

\[
f_3 = -\frac{r^2 + a_{ij}^2 - (z - b_{ij})^2}{(r - a_{ij})^2 + (z - b_{ij})^2}
\]

\[
K(k_{ij}) \quad E(k_{ij})
\]

\[
k_{ij} = \sqrt{\frac{4ra_{ij}}{(r + a_{ij})^2 + (z - b_{ij})^2}}
\]

Complete elliptic integrals of the 1\textsuperscript{st} and 2\textsuperscript{nd} kind with the modulus \( k_{ij} \)
Critical current, Electric field, Voltage

- $I_c = I_c(B, \alpha)$ & $n = n(B, \alpha)$ both are linearized in parts!

- $B_{r,z \ ij} = I_{op} G_{r,z \ ij} + B_{r,z \ ext}$  
  \(\alpha_{ij} = \tan^{-1}\left(\frac{I_{op} G_{r \ ij} + B_{r \ ext}}{I_{op} G_{z \ ij} + B_{z \ ext}}\right)\)

Field magnitude $B_{vij}$

\[
I_{c \ ij} = I_{1 \ ij} + D_{I} I_{1} \left[\sqrt{(I_{op \ r \ ij} + B_{r \ ext})^2 + (I_{op \ z \ ij} + B_{z \ ext})^2} - B_{1}\right]
\]

\[
n_{ij} = n_{1 \ ij} + D_{n} n_{1} \left[\sqrt{(I_{op \ r \ ij} + B_{r \ ext})^2 + (I_{op \ z \ ij} + B_{z \ ext})^2} - B_{1}\right]
\]

Turn in the winding

- $D_{I} = (I_2 - I_1) / (B_2 - B_1)$
- $D_{n} = (n_2 - n_1) / (B_2 - B_1)$

Power law

\[
E_{ij} = E_c \left(\frac{I_{op}}{I_c(B_{vij}, \alpha_{ij})}\right)^{n_{ij}} (B_{vij} \ \alpha_{ij})
\]

\[
V_{ij} = 2 \pi r_{ij} \cdot E_{ij}
\]

- \(V_j = \sum_{i} V_{ij}\)  
- \(V = \sum_{i,j} V_{ij}\)

pancake, coil
Tape - $I_c(B)$: linearized by parts

Coil turn $(i,j)$: load "line"

Examples:
- $G_{rij} = 5 \text{ mT/A}$
- $G_{zij} = 2 \text{ mT/A}$

- $B_{ext} = 0 \text{ T}$
- $B_{zext} = 0 \text{ T}$

- $B_{ext} = 0.5 \text{ T}$
- $B_{zext} = 1 \text{ T}$

$B_v = \sqrt{(I_{op_{rij}} + B_{ext})^2 + (I_{op_{zij}} + B_{zext})^2}$
(A) **Specific case**: Critical current of the turn in coil’s self-field (i.e. $B_{ext} = 0$)

Critical current of the turn $(i,j)$ : $I_{op} = I_{cij}$

The set of 2 linear equations:

1) **Coil**: Turn $(i,j)$ – field magnitude

$$B_{vij} = I_{cij} \cdot G_{vij} = I_{cij} \sqrt{G_{rij}^2 + G_{zij}^2}$$

$$\alpha_{ij} = \tan^{-1} \left( \frac{B_{rij}}{B_{zij}} \right) = \tan^{-1} \left( \frac{I_{cij} \cdot G_{rij}}{I_{cij} \cdot G_{zij}} \right) = \tan^{-1} \left( \frac{G_{rij}}{G_{zij}} \right) \quad \text{- angle}$$

2) **Tape**: $I_{c}(B,\alpha)$ linear in parts

$$I_{cij} - I_{n} = D_{n} \frac{B_{vij}}{B_{n}} - B_{n}$$

$$D_{n} = (I_{n+1} - I_{n})/(B_{n+1} - B_{n})$$

the slope between the two points of the $I_{c}(B)$ characteristic which corresponds to angle $\alpha_{ij}$

$$I_{cij} = \frac{I_{n} - D_{n} \cdot B_{n}}{1 - D_{n} \cdot G_{vij}}$$
(B) **General case**: Critical current of the turn in coil’s self-field + external field (\(B_{\text{ext}}; B_{z\text{ext}}\))

Critical current of the turn (i,j): \(\text{I}_{\text{op}} = I_{cij}\)

The set of 2 equations:

1) **Coil**: Turn (i,j) – field magnitude

\[
B_{r,z \ ij} = I_{cij} G_{r,z \ ij} + B_{r,z \ ext}
\]

\[
\alpha_{ij} = \tan^{-1}\left(\frac{I_{cij} G_{rj} + B_{r\text{ext}}}{I_{cij} G_{zij} + B_{z\text{ext}}}\right)
\]

\[
B_{vij} = \sqrt{(I_{cij} G_{rj} + B_{r\text{ext}})^2 + (I_{cij} G_{zij} + B_{z\text{ext}})^2}
\]

2) **Tape**: \(I_{c}(B,\alpha)\) linear in parts

\[
I_{cij} - I_n = D_n (B_{vij} - B_n)
\]

where \(D_n = (I_{n+1} - I_n)/(B_{n+1} - B_n)\)

Final non-linear (quadratic) equation with respect to \(I_{cij}\)

\[
I_{cij} - I_n = D_n \left( (I_{cij} G_{rj} + B_{r\text{ext}})^2 + (I_{cij} G_{zij} + B_{z\text{ext}})^2 - B_n^2 \right)
\]

The most simple way to solve: **iterative procedure** - increase of \(\text{I}_{\text{op}}\) in small steps \(\delta\text{I}_{\text{op}}\)
Model coil specification

- Inner winding radius \( a_1 \) [mm] = 27.5
- Outer winding radius \( a_2 \) [mm] = 59
- Coil length \( 2b \) [mm] = 69
- Number of pancakes \( N_z \) = 22
- Number of turns per pancake \( N_r \) = 48
- Overall turns number \( N \) = 1056
- Self-inductance \( L \) [mH] = 54.9
- Critical current @ 20 K \( I_{cmin} \) [A] = 101.1 A

Fictitious winding

(\( N_b = 22 \))

(\( N_a = 24 \))
Distribution of the magnetic field in the coil winding @ different Bzext

Iop = 103 A ; Icmin = min(Ic) = 101.1 A ; i.e. Iop > Icmin

The length of the magnetic flux density vectors is normalized to respective maximum value of the field

<table>
<thead>
<tr>
<th>Bzext (T)</th>
<th>0 - 1.35 T</th>
<th>0.76 - 2.35</th>
<th>2.75 - 4.35</th>
<th>9.76 - 11.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bv (T)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>max (α)</td>
<td>90</td>
<td>24</td>
<td>9.5</td>
<td>3.1</td>
</tr>
<tr>
<td>[deg]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>max (Br)</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>[T]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The length of the magnetic flux density vectors is normalized to respective maximum value of the field
Magnetic flux density in the winding – absolute value $B_v(i,j)$

Magnetic field orientation – angle $\alpha(i,j)$
Winding: Field orientation vs field magnitude

\[ \alpha_{ij} [\text{deg}] \]

\[ B_{z_{\text{ext}}} [\text{T}] \]
- \( \bigcirc \) 0
- \( \Delta \) 1
- \( \square \) 3

\[ I_{\text{op}} = 103 \text{ A} \]
Winding: Correlation between the field orientation ($\alpha$) and $\text{abs}(Br)$
Critical currents $I_c(i,j)$ of individual turns

- **Bzext = 0**
  - No electric field
  - $I_c(i,j)$ decreases
  - $E(i,j)$ decreases

- **Bzext = 1 T**
  - $I_c(i,j)$ increases
  - $E(i,j)$ increases

- **Bzext = 3 T**
  - $I_c(i,j)$ decreases
  - $E(i,j)$ decreases
  - Electric field area spreads over the whole winding

Only small differences in $I_c(i,j)$
Critical current $I_{c(i,j)}$ vs magnetic field magnitude $B_{v(i,j)}$.

$I_{op} = 103$ A

Short sample:
- $\star$ 0 deg
- $\times$ 30 deg
- $\times$ 60 deg
- $\star$ 90 deg

$B_{zext}$ [T]
- 0 deg
- 30 deg
- 60 deg
- 90 deg
- 10 deg
Critical current $I_c(i,j)$ vs magnetic field orientation $\alpha(i,j)$

$I_{op} = 103$ A

$B_{zext} [T]$
- $0$
- $1$
- $3$
- $10$

$\alpha_{ij} [\text{deg}]$
Critical current $I_{c(i,j)}$ vs radial component of the magnetic field $B_{r(i,j)}$

$I_{op} = 103$ A

$B_{zext}$ [T]
- $0$
- $1$
- $3$
- $10$

$|B_r|$ [T]
Critical current $I_{c(i,j)}$ vs radial magnetic field $B_{r(i,j)}$ - detail

$I_{op} = 103$ A
$B_{zext} = 0$ T

$I_{op} = 103$ A
$B_{zext} = 1$ T

$I_{op} = 103$ A
$B_{zext} = 3$ T

$I_{op} = 103$ A
$B_{zext} = 10$ T
Eij vs Bv

Eij vs α

Eij vs abs(Br)
Electric field $E(i,j)$ vs radial magnetic field $Br(i,j)$ - detail

- $I_{sp} = 103$ A
  $B_{zext} = 0$ T

- $I_{sp} = 103$ A
  $B_{zext} = 1$ T

- $I_{sp} = 103$ A
  $B_{zext} = 3$ T

- $I_{sp} = 103$ A
  $B_{zext} = 10$ T
Individual turns - Correlation between electric field $E(i,j)$ and critical current $I_{c(i,j)}$
Voltage on individual pancakes
parameter : $B_{\text{zext}}$
Critical current of the coil $I_{cmin}$ @ different operating currents $I_{op}$

$I_{cmin} = \text{min}(I_{cij})$

Under-critical state:
$I_{op} < I_{cmin}$

Over-critical state:
$I_{op} < I_{cmin}$
Voltage on coil terminals $U$ as a function of $B_{zext}$

Parameter: operating current $I_{op}$

Under-critical state
$I_{op} < I_{cmin}$

Over-critical state
$I_{op} < I_{cmin}$
Critical current of the coils $I_{cmin}$ vs operating current $I_{op}$

$B_{zext}$ [T]:
- $0$
- $0.5$
- $1$
- $1.5$
- $2$
- $3$
- $4$
- $5$
- $10$

Under-critical state: $I_{op} < I_{cmin}$

Over-critical state: $I_{op} > I_{cmin}$
V(I) characteristic of the coil
Parameter : Bzext
\( I_{cmin} = f(I_{op}, B_{zext}) \)

Position of the weak turn in the winding
CONCLUSIONS

Factors limiting both:
- the critical current of the individual turn $I_{cij}$
- the critical current $I_{cmin} = \min(I_{cij})$ of the HTS coil:

- Anisotropy in the $I_c(B)$ of the tape; magnetic field amplitude + field orientation
- $I_{cij}$ – is not determined exclusively by the radial component of the magnetic field. It is a compromise between the field magnitude and the field orientation.

Weak turns of the HTS cylindrical coil with uniform winding structure are always located in the outermost pancakes. Their accurate position in the pancakes depends on the winding geometry and external field. In case of high field insert coils it is the mid-layer.

Numerical simulations predict rather unexpected behaviour of the HTS coils if an external magnetic field $B_{zext}$ parallel with the coil axis is applied:

- High magnetic loading (field magnitude) of weak turns is compensated by the decrease of an angle between the tape surface and field orientation.

Consequence:
- increase in the critical current $I_{cmin}$ at low external fields $B_{zext}$
- decrease of the voltage $U$ on magnet terminals
The anisotropy in \( I_c(B) \) is not applied / does not play any role at high \( B_{\text{ext}} \) values.

Consequence:

In case of the HTS insert coils it is necessary to concentrate rather on the production of the tapes with the current carrying capacity as high as possible than on the attempt to decrease the anisotropy in \( I_c(B) \).

... the experience shows that the decrease of the \( I_c(B) \) anisotropy by changing the filament architecture in the tape is always accompanied by a simultaneous decrease of the current carrying capacity ...

*The above results were published in:*  