Mathematical modeling of two parallel Josephson junctions stacks

I. R. Rahmonov\textsuperscript{a, b}, Yu. M. Shukrinov\textsuperscript{a, b, c}, P. Seidel\textsuperscript{d}, M. Grajcar\textsuperscript{e}, A. Plecenik\textsuperscript{f} and W. Nawrocki\textsuperscript{g}

\textsuperscript{a}BLTP, Joint Institute for Nuclear Research, Dubna, Russia
\textsuperscript{b}UMAROV Physical Technical Institute, Dushanbe, Tajikistan
\textsuperscript{c}Dep. of Theoretical Physics, International University of Dubna, Dubna, Russia
\textsuperscript{d}Institut für Festkörperphysik, Friedrich-Schiller-Universität Jena, Jena, Germany
\textsuperscript{e}Department of Experimental Physics, Comenius University, Bratislava, Slovakia
\textsuperscript{f}Poznan University of Technology, Poznan, Poland

\textbf{Abstract}

We derived the system of nonlinear differential equations for describing the phase dynamics of two parallel stacks of JJ in the framework of the CJCJ+DC model. Using the fourth order Runge-Kutta method we solve this system of equations and calculate the total current voltage characteristics (CVC) for the system with different $N_1$ and $N_2$. Superconducting, quasiparticle, displacement and diffusion currents and CVC for each stack are described in detail. Analysis of the CVC shows the features related to the transitions between the states with different number of rotating and oscillating junctions. Transition in one stack stimulates the corresponding transition in another one.

\textbf{Model and numerical method}

System of the normalized equations, which describe above scheme, can be written as \cite{1, 2}

\begin{equation}
\begin{aligned}
\dot{\psi}_m &= U_m - \alpha_1(U_{m+1} + U_{m-1} - 2U_m) \\
\dot{\varphi}_m &= V_m - \alpha_2(V_{m+1} + V_{m-1} - 2V_m) \\
U_m &= \frac{1}{\mathcal{C}} (I - \beta \sin \psi_m - \mu \psi_m - V_m - \alpha \varphi_m - \beta \varphi_m) \\
V_m &= \sum_{k=1}^{N_2} \left( I - \sin \varphi_k - \beta \varphi_k - \mu \varphi_k \right) - \\
& - \sum_{k=1}^{N_1} \left( I - \sin \psi_k - \mu \psi_k \right)
\end{aligned}
\end{equation}

where $\psi_m$ and $U_m$ are phase differences and voltage of $m$th JJ in the first stack, $\varphi_m$ and $V_m$ are phase differences and voltage of $m$th JJ in the second stack. $N_1, N_2$ number of JJ, $\alpha_1, \alpha_2$ coupling parameters, $\mu, \beta$ dissipation parameters respectively for the first and second stacks. Voltage $V_m$ and $U_m$ are normalized to $V_m = \frac{I_{J1}}{R_{J1}}$ and $U_m = \frac{I_{J2}}{R_{J2}}$ bias current $I$ to critical current $I_{c1}$ of JJ of first stack. time $t$ to plasma frequency $\omega_p = \sqrt{\frac{2e}{\mu_0 \mathcal{C}}}$. The ratio of critical current, resistance and capacitance of JJs of stack 1 and 2. The dissipation parameter is $\beta = \frac{1}{R_1} \sqrt{\frac{2e}{\mu_0 \mathcal{C}}}$. 

The matrix $B$ have a form as

\begin{equation}
B = \begin{pmatrix}
C + N_1 & C & \cdots & C \\
C & C + N_1 & C & \cdots & C \\
& \vdots & \ddots & \ddots & \vdots \\
C & \cdots & C & C + N_1 & C \\
C & \cdots & C & C & C + N_1 \\
\end{pmatrix}
\end{equation}

Using the 4th order Runge-Kutta method we solve above system of equations at fixed value of external current $I$. in time domain $[0, T_{max}]$ and find phase differences $\dot{\varphi}$ and Voltage $V$ as function of time. Averaging on the time domain Voltage we find one point of CVC. Increasing $I$ to $4I$ process repeats. External current increases till $I_{max}$ and decreases to zero.

\textbf{CVC of the two parallel JJ stacks}

Fig. 1 Scheme of the parallel connection of the two stacks of JJs. Superconductor-insulator-superconductor (SIS) model.

Fig. 2 CVC of the two parallel JJ stacks: (a) for $N_1 = 3, N_2 = 3$; (b) for $N_1 = 1, N_2 = 3$ by the increasing of $I$; (c) for $N_1 = 1, N_2 = 3$ by the decreasing of $I$.

\textbf{Dependence of average currents on the bias current}

Fig. 3 Average superconducting, quasiparticle, diffusion and displacement currents depending on the bias current for the second stack by increasing and decreasing of bias current.

Fig. 4 Average superconducting, quasiparticle and displacement currents depending on bias current for first stack by increasing and decreasing of bias current.

\textbf{Time dependence of the currents}

Fig. 5 Charge-time dependence in the superconducting layers for the second stack at $I = 1.54$.

\textbf{Time dependence of the electric charge in the superconducting layers}

\textbf{Conclusions}

1. We found that when the current through the one stack reaches the critical value, all JJs go to the rotating state for short time and then some of them return back to the oscillating state. In this region of CVC charge density wave is forming along the stack forms.

2. In the hysteresis region of CVC we have observed an unstable rotating states. Such behavior is not observed in the case of single stack.

\textbf{References}

\cite{1} I. R. Rahmonov and Yu. M. Shukrinov, Particle and Nucl. Lett. 11, 6 (2014) accepted.