Modeling superconductor-chip-based magnetic traps for ultra-cold atoms

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Magnetic traps for cold atoms

An area of research where solid-state, atom and optic physics meet.

Perez-Rios and Sanz (2013):
"Magnetic trapping is a cornerstone for modern ultracold physics and its applications (e.g., quantum information processing, quantum metrology, quantum optics, or high-resolution spectroscopies)."

Cano et al. (2008):
"... these traps play a fundamental role in studies of atom-surface interactions (the Casimir-Polder force), the spin decoherence of atoms near dielectric bodies, and in the usage of trapped atoms to probe local irregularities of magnetic and electric fields near conductive films".

Atom traps are a tool for study, e.g.:
Bose-Einstein condensation; coherence and decoherence processes,

and can be used :
for creation of atom SQUIDs and interferometers, atom clocks and lasers.

Magnetic traps for cold atoms

A **magneto-optical trap** (**MOT**) is a device that uses laser cooling with magneto-optical trapping in order to produce clouds of cold, trapped, neutral atoms at temperatures as low as several microkelvins or tens nanokelvins.

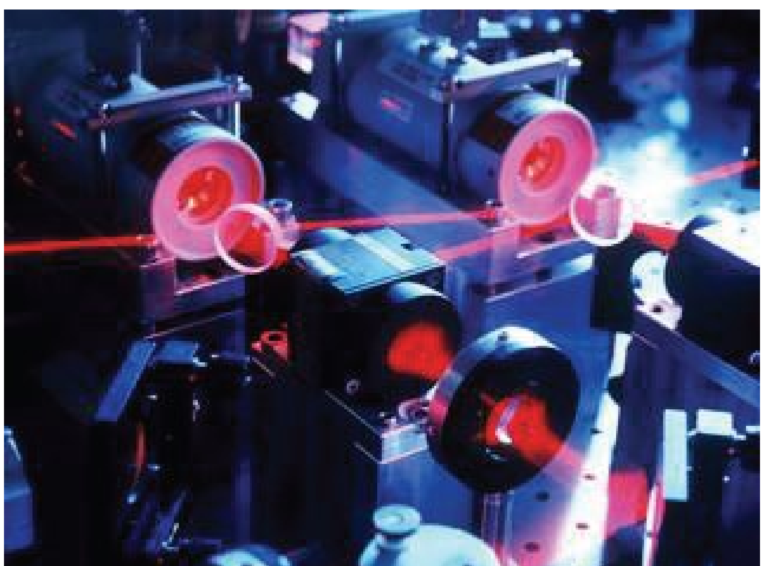
A **magnetic trap** is an apparatus which uses a magnetic field gradient to trap neutral particles with nonzero magnetic moments.

Laser cooling

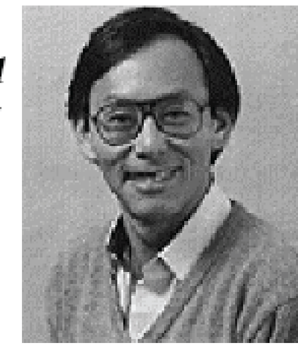
Nobel Prize 1997

„for development of methods to cool and trap atoms with laser light“

- Doppler cooling (Doppler cooling limit ~200 μK);
- Sisyphus cooling (limit 0.2-2.4 μK);
- Doppler effect and dark state ~ 100 nK;
- Resolved sideband cooling.



Steven Chu
Stanford University

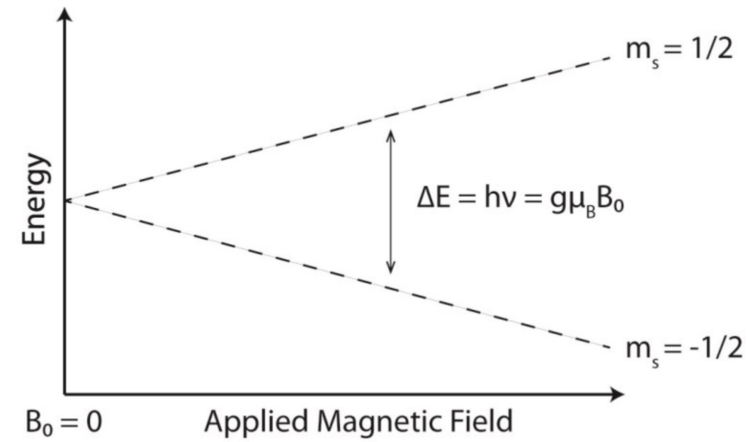


Claude Cohen-Tannoudji
Collège de France und
École Normale Supérieure

William D. Phillips
National Institute of Standards and Technology

Magnetic trap

The Zeeman effect, named after the Dutch physicist Pieter Zeeman, is the effect of splitting a spectral line into several components in the presence of a static magnetic field.



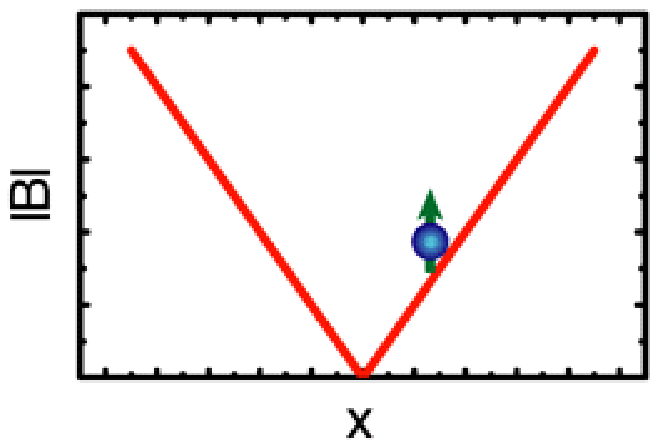
Earnshaw's theorem

There can be no isolated maximum of $|B|$ in the free space (only at the sources of the field):

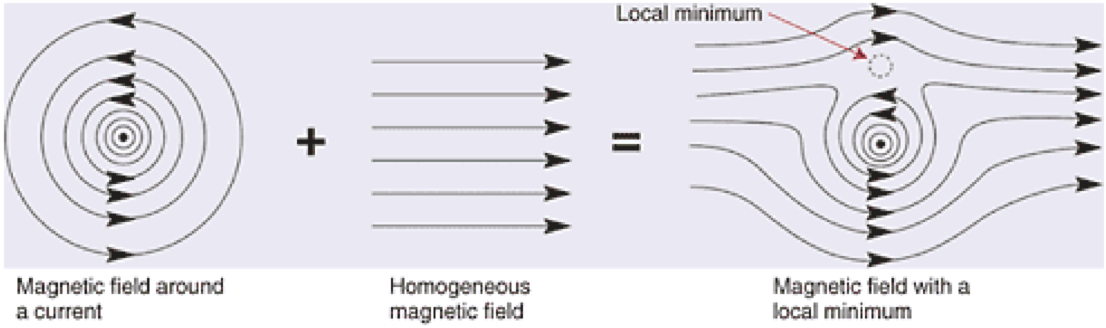
$$\Delta |B|^2 = 2\sum_{i=1}^{3}\left(B_i \Delta B_i + |\nabla B_i|^2\right) = 2\sum_{i=1}^{3}|\nabla B_i|^2 \geq 0$$

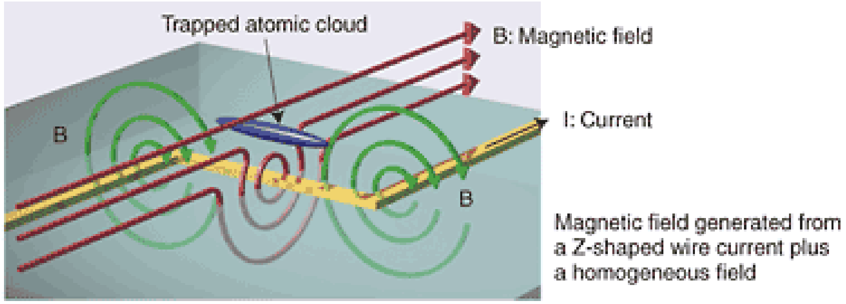
An isolated minimum can be created.

Adiabatic approximation

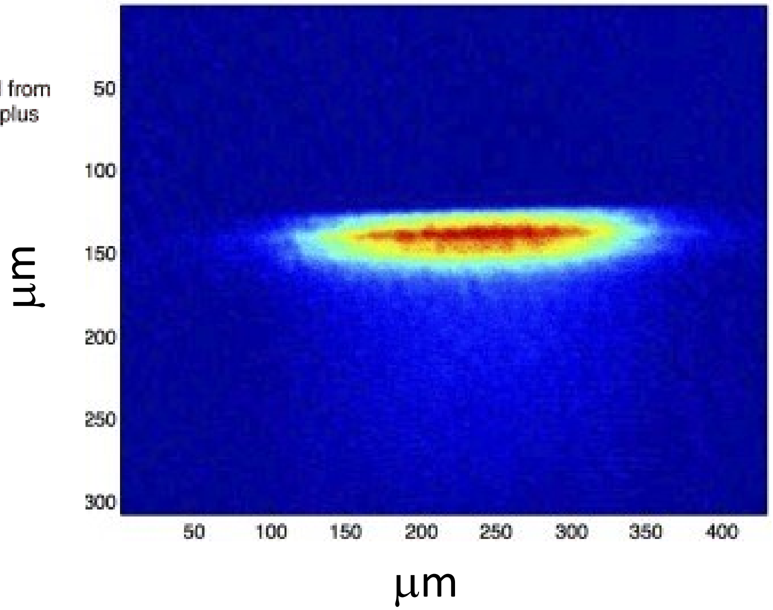


Simple magnetic trap

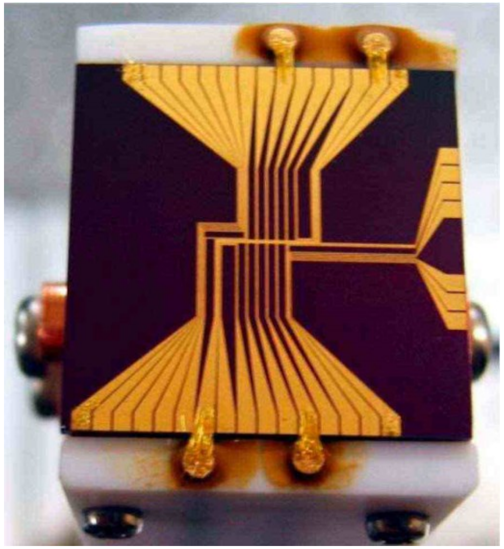




Atom cloud



Advantages of a superconductor chip



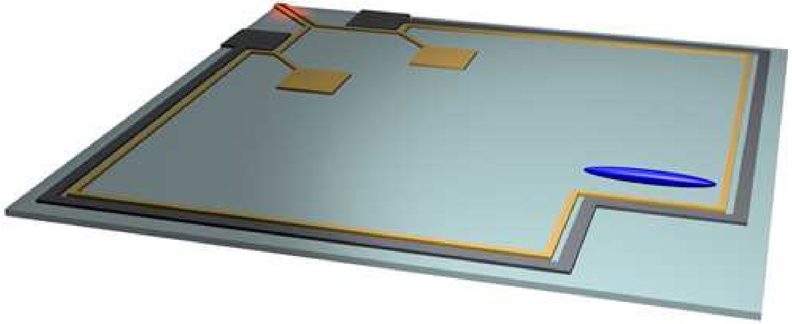
Main disadvantages of normal metal chips

Current-induced thermal magnetic noise and technical noise harmfully influence the atom cloud and dominantly limit its lifetime especially near the chip surface, at trap height < 10-20 μm.

Advantage of a superconductor chip

is a significant enhancement of trapping lifetime due to a strong reduction of

- electromagnetic noise near the film surface;
- technical noise using persistent currents and frozen magnetic flux.



Superconductors are expected to play an essential role in this emerging field of research because they can provide an extremely low noise environment for trapped atoms.

In most experiments ^{87}Rb atoms in the state $|F=2,\ m_f=2\rangle$ are trapped.

Task

Several traps created by a magnetic field of transport or persistence currents as well as by frozen magnetic flux in a superconductor, supplemented by a bias field, have been experimentally investigated.

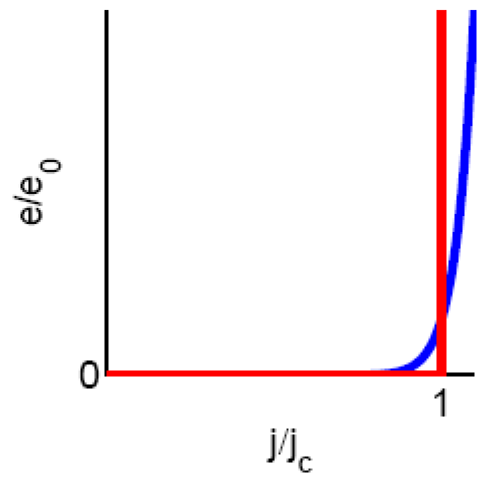
Although thin superconducting films of different shapes have been employed, numerical and analytical simulations of the magnetic fields were developed only for simple shapes for which the current density distribution is one-dimensional: an infinite strip, disk, and ring.

Here we report a new method and results of numerical simulation of the three-dimensional magnetic trap created using a chip based on a superconducting thin film of arbitrary shape.

Two stages of calculation:

- 2D current distribution in a thin type-II superconductor film in the mixed state is computed;
- 3D magnetic trap is calculated using the Biot-Savart law.

Model and numerical approach



The film is thin: the current density is replaced by the sheet current density.

Power relation between the component of the electric field tangential to the film, $\boldsymbol{e}$, and sheet current density $\boldsymbol{j}$:

$$\boldsymbol{e} = e_0(|\boldsymbol{j}|/j_c)^{p-1}\boldsymbol{j}/j_c$$

The critical-state model: the $p \to \infty$ limit of the power law (Bean 64).

To solve the magnetization and transport current problems for flat films of arbitrary shapes, including multiply connected films, we used the numerical method based on a new variational formulation written for two-variables, the electric field and magnetization function, and a non-conforming numerical approximation

J. W. Barrett, L. Prigozhin, V. Sokolovsky, Superconduct. Sci. Technol. 26, 105009, 2013

Dimensionless form

In simulation we used the Bean model and the infinitely thin film approximation.
The magnetic field is found using the Biot-Savart law. In dimensionless variables $\mathbf{b} = \mathbf{B}/\mu_0 J_c$, $\mathbf{b}_{ext} = \mathbf{B}_{ext}/\mu_0 J_c$, $\mathbf{j} = \mathbf{J}/J_c$, $\mathbf{r} = \mathbf{R}/w$ it reads

$$\mathbf{b} = \mathbf{b}_{ext} + \frac{1}{4\pi}\int_{\Omega} \frac{\mathbf{j}\times\hat{\mathbf{r}}}{r^2} d\Omega$$

where $\mathbf{B}_{ext}$ is the external field, $\hat{\mathbf{r}}$ is the unit vector, w is the characteristic film size.

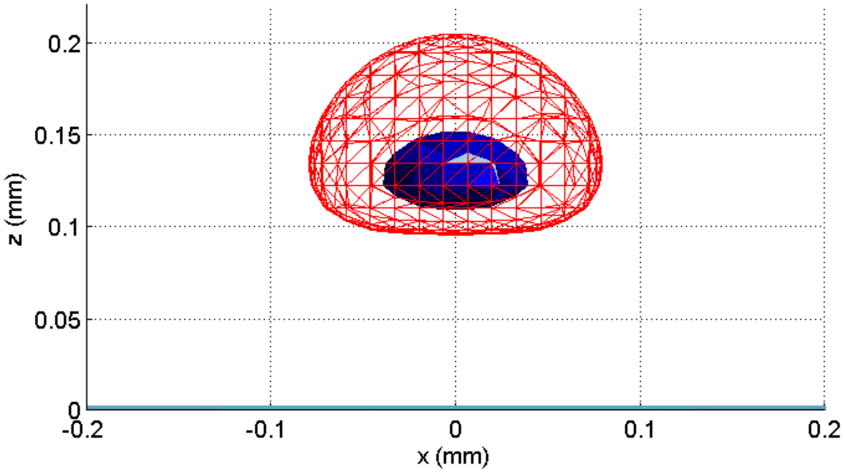
The normalized magnetic field depends neither on the critical current density J_c nor on film size w and can be presented as

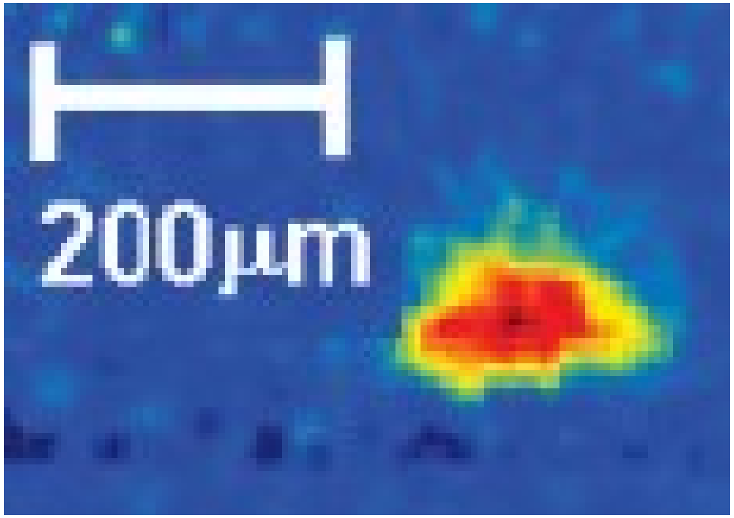
$$\mathbf{b} = \frac{\mathbf{B}}{\mu_0 J_c} = \mathbf{\Phi}\left[\frac{\mathbf{r}}{w}, \frac{\mathbf{B}_{ext}}{\mu_0 J_c}, \frac{I}{J_c w}\right]$$

where the functional $\mathbf{\Phi}$ takes into account the history of variations of the transport current and external magnetic field.

Magnetic trap on a square chip

Two pulses of the external field: $0 \to 3\mu_0 J_c \to 0$ and $0 \to -0.8\mu_0 J_c \to 0$





a)

b)

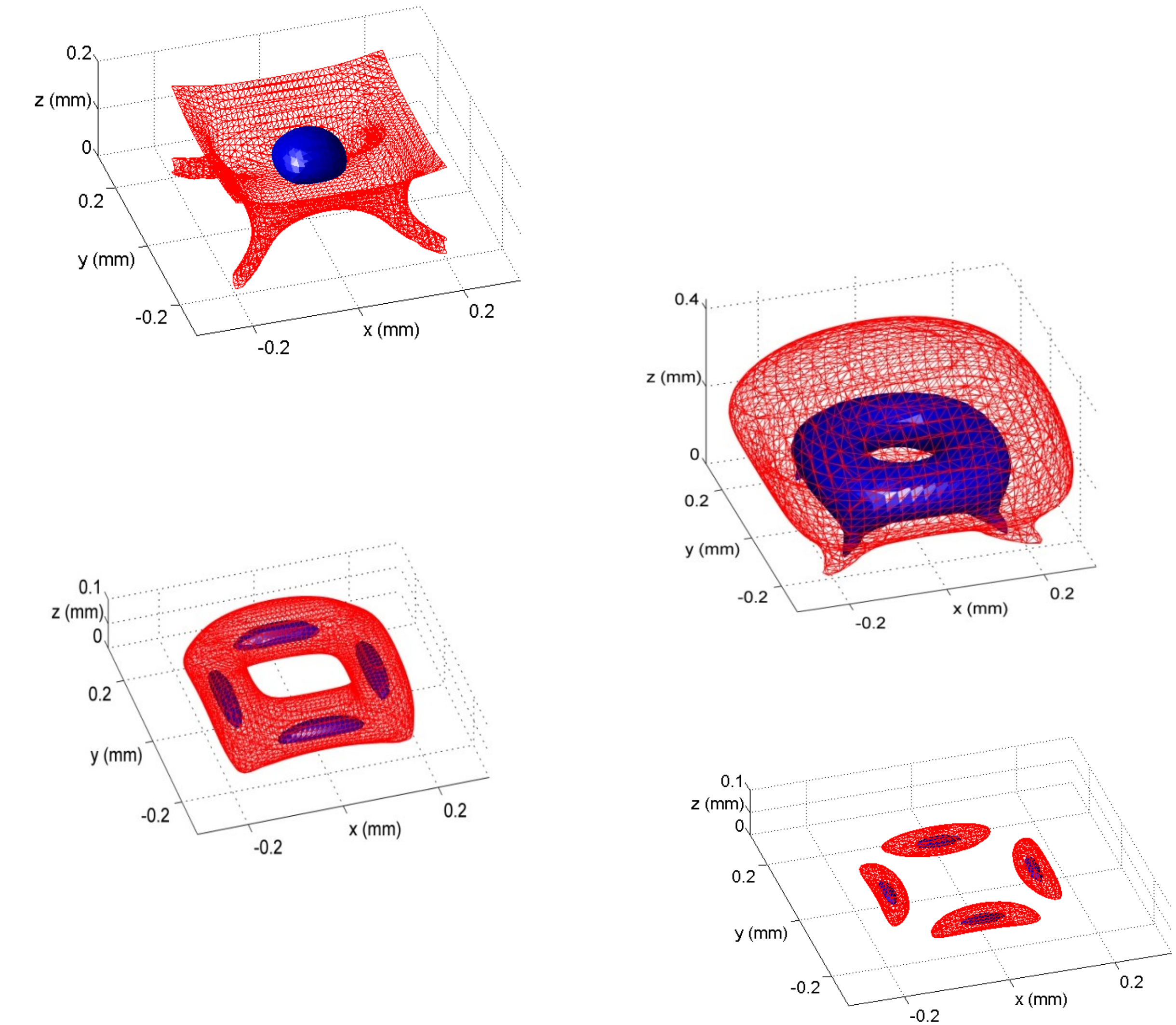
a) computed magnetic field level surfaces, $|B|=0.03\mu_0 J_c$ (blue solid surface) and $|B|=0.06\mu_0 J_c$ (red lines);

b) the atom cloud image; taken from Fig. 4 in M. Siercke, et al. *Phys. Rev. A*, **85**, 041403 (R) (2012)

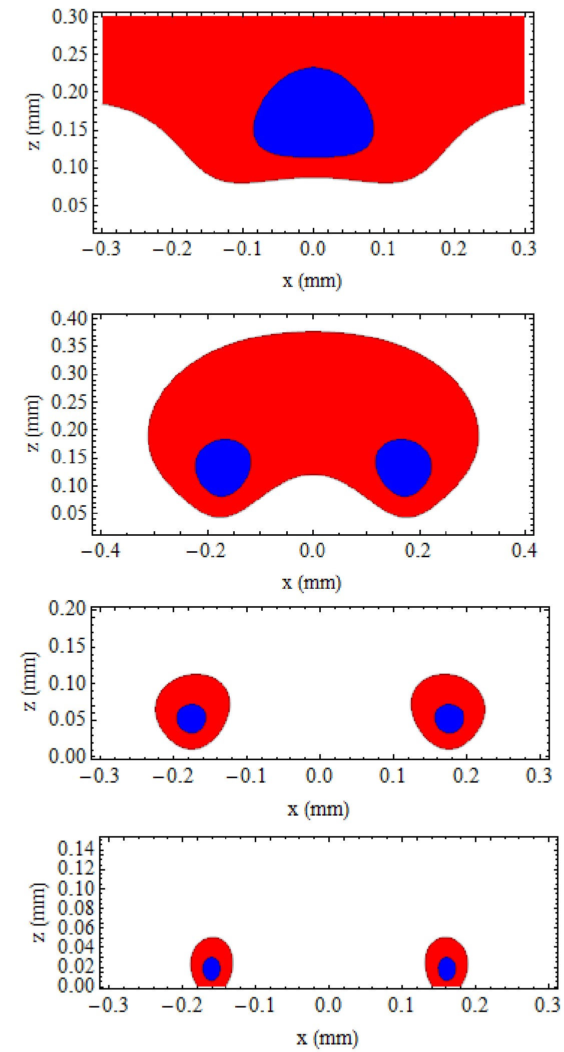
Square chip: bias field influence

Two pulses of the external field: $0 \rightarrow 3\mu_0 J_c \rightarrow 0$ and $0 \rightarrow -0.8\mu_0 J_c \rightarrow 0$

The applied bias field $B_{bias} / \mu_0 J_c$ values (from top to bottom): 0, 0.2, 0.4, 0.6 .



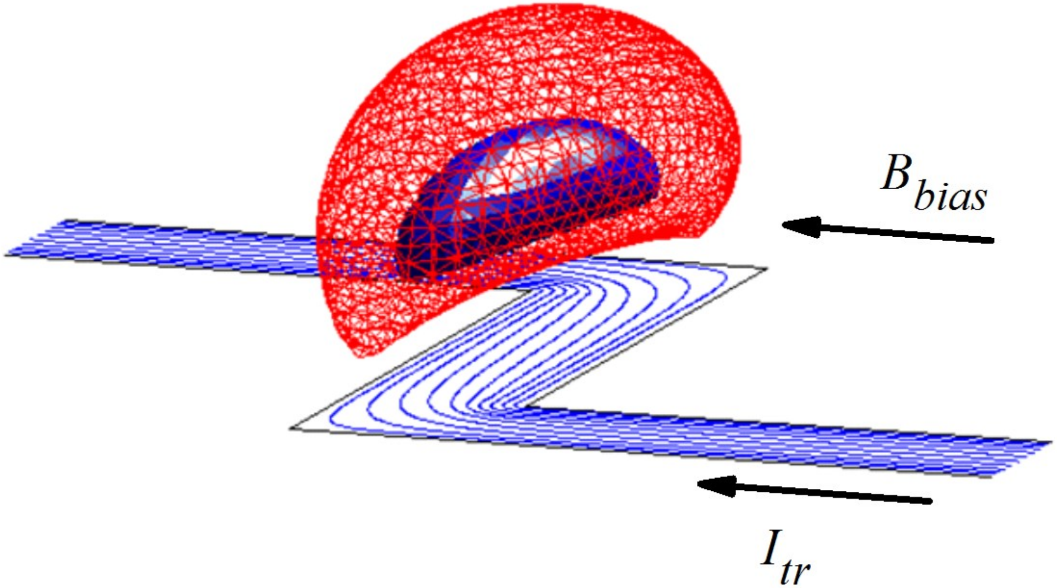
Disc

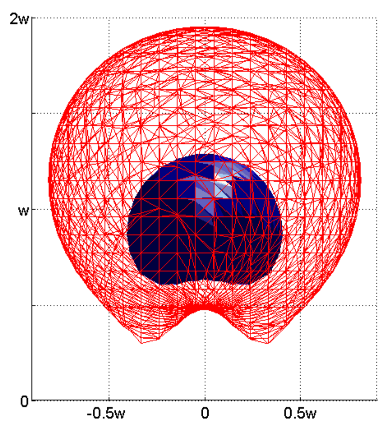


$| B | / \mu_0 J_c = 0.12$ - red

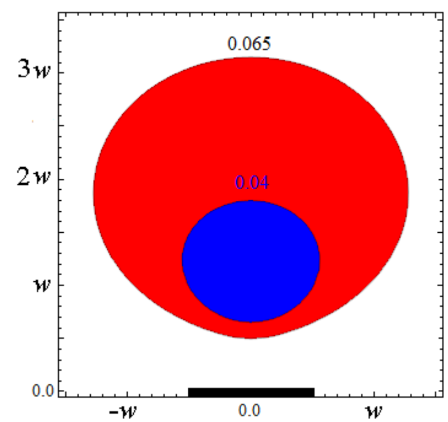
$| B | / \mu_0 J_c = 0.05$ - blue

Z-chip: transport current





Infinitely long strip

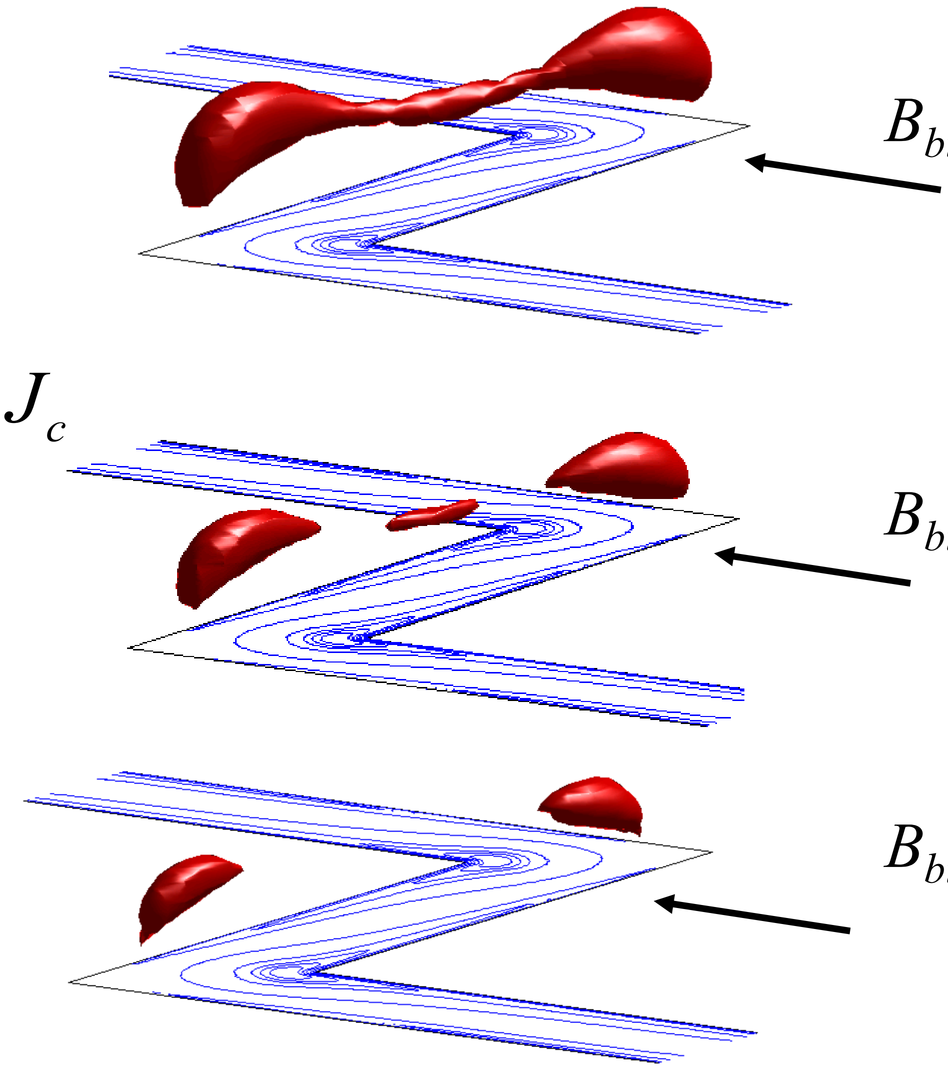


Transport current $I_{tr}: 0 \longrightarrow 0.7I_c$; $B_{bias} = 0.1\mu_0 J_c$

Level surfaces: $|B| = 0.04\mu_0 J_c$ and $|B| = 0.065\mu_0 J_c$

Z-chip: current pulse $0 \rightarrow 0.7 I_c \rightarrow 0$

$|B| = 0.004 \mu_0 J_c$



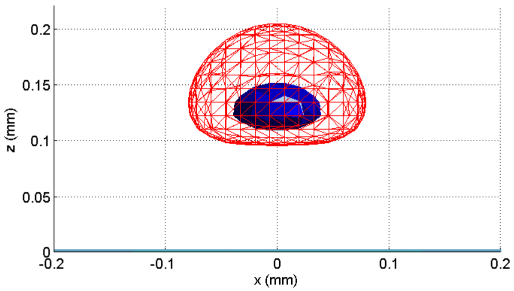
$B_{bias} = 0.007 \mu_0 J_c$

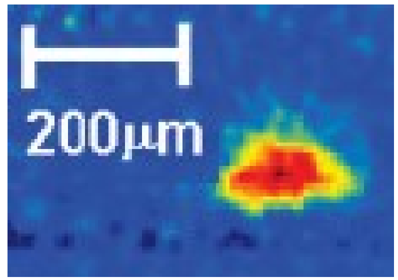
$B_{bias} = 0.008 \mu_0 J_c$

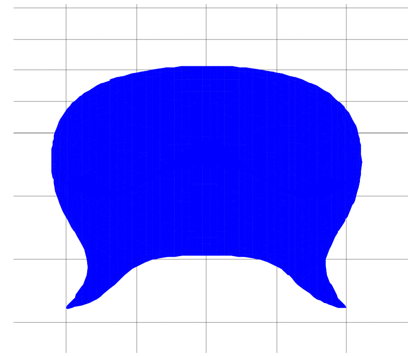
$B_{bias} = 0.01 \mu_0 J_c$

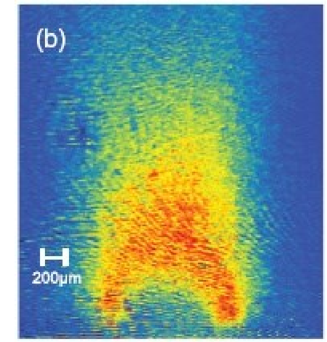
Interferometers can be built in the temporal domain

Comparison of Experiment and Theory









Theory: levels of equal magnetic field magnitude are presented.

Experiment: the observed atom cloud shape is also influenced by
- Direction of observation;
- Thermal distribution of atoms in a trap;
- Inhomogeneity of the superconductor;
- Gravitaty and other forces;
- Loss of atoms due to their leaving the trap;
- Finite thickness of superconducting film.

Stability of trapping

Criteria of atom trapping stability :

- Trap depth

$B_{dep} \geq 10 k_B T/\mu$,

where T is the atom gas temperature, k_B is the Boltzmann constant, μ is the atom magnetic moment,
B_{dep} is the trap depth, the difference between the maximal level of the magnetic field magnitude, $|\boldsymbol{B}|$, for which the iso-surface is closed, and its minimum in the trap.

- Magnetic field gradient

should be high enough to protect the atoms from gravity's pull.

For ^{87}Rb atoms at 1 μK with $\mu = 2$:
trap depth > 0.07 G;
field gradient > 15.3 G/cm.

Superconducting film characteristics

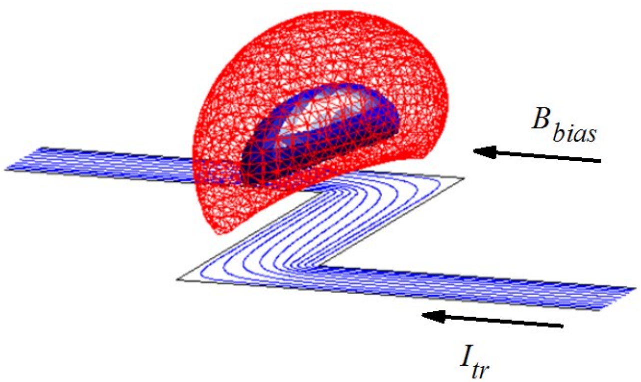
Super-conductor	Temperature K	Thickness, μm	J_c, A/m	$\mu_0 J_c$, G
Nb	4-6	0.4-0.9	$(1.6\div3.6)\times10^4$	200÷400
MgB_2	4	1.6	1.6×10^5	2000
YBCO (typical)	77 77 83	0.3 0.6-0.8 0.3	1.1×10^4 $(1.2\div2.1)\times10^4$ 0.4×10^4	140 150-260 50
Ag doped YBCO multilayered structure	77 10	1 1	3×10^4 30×10^4	380 3800

The trap size is proportional to the characteristic size w of the film. The strip width in the superconducting chips is within the range from 10 mm till 300 mm; half square side is 500 mm.

B_{dep} is proportional to J_c; the field gradient is $\sim B_{dep}/w \sim J_c/w$

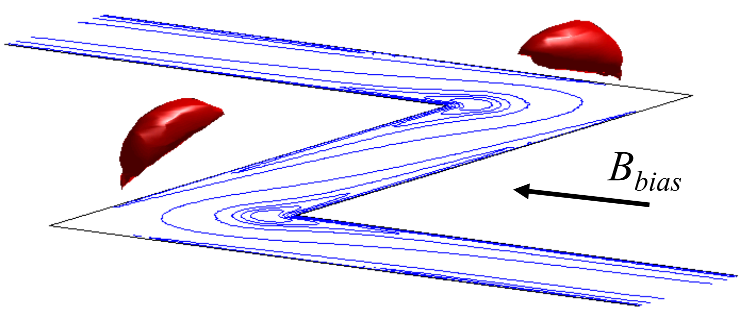
Analysis of trapping stability

Requirements: trap depth > 0.07 G; field gradient > 15.3 G/cm.



Z-shaped MgB_2 chips:
$\mu_0 J_c$=2000 G; w =100 μm,
transport current - 0.7 I_c,
closed level 0.065$\mu_0 J_c$

trap depth 0.044$\mu_0 J_c$ =88 G;
field gradient ~8800 G/cm.



current pulse 0→0.7 I_c → 0

trap depth 0.002 $\mu_0 J_c$= 4 G
field gradient >400 G/cm.

Hence, such a superconducting chip ensures stable magnetic trapping of could atoms.

Conclusion

The developed method allows one to simulate the 3D magnetic traps on superconducting chips. The method is based on numerical solution of transport current and/or magnetization problems for thin superconducting films of arbitrary shape. Both the Bean and the power law current-voltage relations can be used.

Our simulations have been performed for the chip configurations employed in recent cold atom experiments. The developed approach takes into account the superconductor properties and variation of the external magnetic field and transport current and enables one to analyze such important characteristics of the magnetic traps as their depth, size, shape, and distance from the chip surface. Knowledge of these characteristics is important for designing a cold atom physics experiment.

Thank you for your attention